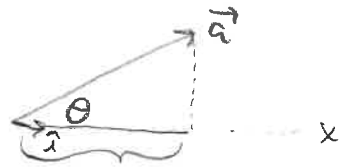
Tues 8/26

Projections

Consider $\vec{a} = \langle a_1, a_2 \rangle$.



$$a_1 = |\vec{a}| \cos\theta$$

Here a_1 is the scalar projection of $\vec{a}$ onto $\hat{\imath}$, written

$$\text{comp}_{\hat{\imath}}\, \vec{a} = \vec{a} \cdot \hat{\imath} = |\vec{a}||\hat{\imath}|\cos\theta = |\vec{a}|\cos\theta = a_1$$

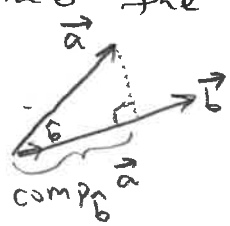
We could also do

$$\text{comp}_{\hat{\jmath}}\, \vec{a} = \vec{a} \cdot \hat{\jmath} = \langle a_1, a_2 \rangle \cdot \langle 0, 1 \rangle = a_2$$

The vector projection of $\vec{a}$ onto $\hat{\imath}$ is

$$\text{proj}_{\hat{\imath}}\, \vec{a} = \text{comp}_{\hat{\imath}}\, \vec{a} = (\vec{a} \cdot \hat{\imath})\, \hat{\imath}$$

Now find the scalar projection of $\vec{a}$ onto $\vec{b}$.



$$\text{comp}_{\hat{b}}\, \vec{a} = \vec{a} \cdot \hat{b} = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|}$$

And the vector projection is

$$\text{proj}_{\hat{b}}\, \vec{a} = (\vec{a} \cdot \hat{b})\, \hat{b} = \frac{(\vec{a} \cdot \vec{b})\, \vec{b}}{|\vec{b}|^2}$$

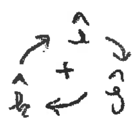
§3.4 Cross Product

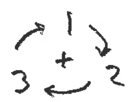
Let $\vec{a} = \langle a_1, a_2, a_3 \rangle$ and $\vec{b} = \langle b_1, b_2, b_3 \rangle$ are two vectors. the cross product is defined as

$$\vec{a} \times \vec{b} = (a_2 b_3 - a_3 b_2)\hat{i} + (a_3 b_1 - a_1 b_3)\hat{j} + (a_1 b_2 - a_2 b_1)\hat{k}$$

Example: Let $\vec{a} = \langle 2, -1, 1 \rangle$ and $\vec{b} = \langle 3, 2, -4 \rangle$

$$\vec{a} \times \vec{b} = (-1(-4) - 1(2))\hat{i} + (1(3) - 2(-4))\hat{j} + (2(2) + 1(3))\hat{k}$$
$$= 2\hat{i} + 11\hat{j} + 7\hat{k}$$





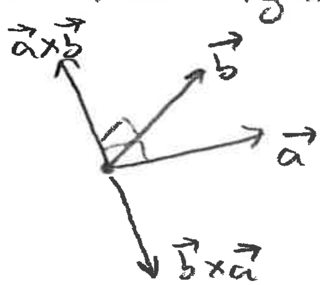
Example: Find $(\vec{a} \times \vec{b}) \cdot \vec{a}$.

Solution: $(\vec{a} \times \vec{b}) \cdot \vec{a} = \langle a_2 b_3 - a_3 b_2, a_3 b_1 - a_1 b_3, a_1 b_2 - a_2 b_1 \rangle \cdot \langle a_1, a_2, a_3 \rangle$

$$= a_2 b_3 a_1 - a_3 b_2 a_1 + a_3 b_1 a_2 - a_1 b_3 a_2 + a_1 b_2 a_3 - a_2 b_1 a_3$$

$$= 0$$

So we see that $\vec{a} \times \vec{b}$ is orthogonal to $\vec{a}$. Similarly, we can show that $(\vec{a} \times \vec{b}) \cdot \vec{b} = 0$. So $\vec{a} \times \vec{b}$ is orthogonal to both $\vec{a}$ and $\vec{b}$, and it has direction according to the right hand rule.



$$\vec{a} \times \vec{b} = -(\vec{b} \times \vec{a})$$

$$\vec{b} \times \vec{a} = (b_2 a_3 - b_3 a_2)\hat{i} + (b_3 a_1 - b_1 a_3)\hat{j} + (b_1 a_2 - b_2 a_1)\hat{k}$$
$$= (2(1) - (-4)(-1))\hat{i} + (-4(2) - 3(1))\hat{j} + (3(-1) - 2(2))\hat{k}$$
$$= -2\hat{i} - 11\hat{j} - 7\hat{k} = -(2\hat{i} + 11\hat{j} + 7\hat{k}) = -(\vec{a} \times \vec{b})$$