Fri 8/22

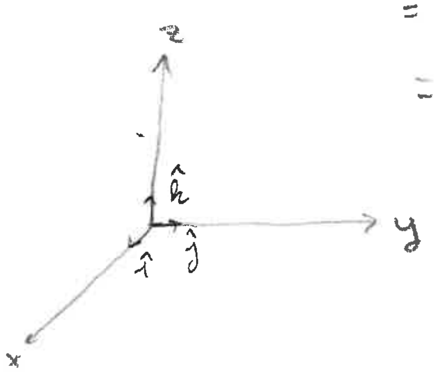
§2.2 Vectors
- unit vectors
- difference of vectors
- Vector space
- Application (Newton's Law)

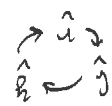
§2.3 Dot Product
- Definition
- Properties
  - orthogonality
  - parallel
  - Cauchy-Schwarz Inequality
  - Triangle Inequality

$$\frac{\vec{v}}{|\vec{v}|} = \hat{v} \qquad |\hat{v}| = 1$$

$$\hat{i} = \langle 1, 0, 0 \rangle$$
$$\hat{j} = \langle 0, 1, 0 \rangle$$
$$\hat{k} = \langle 0, 0, 1 \rangle$$

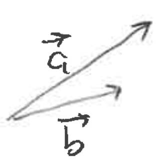
$$\vec{v} = \langle -2, 4, 7 \rangle = \langle -2, 0, 0 \rangle + \langle 0, 4, 0 \rangle + \langle 0, 0, 7 \rangle$$
$$= -2\langle 1, 0, 0 \rangle + 4\langle 0, 1, 0 \rangle + 7\langle 0, 0, 1 \rangle$$
$$= -2\hat{i} + 4\hat{j} + 7\hat{k}$$



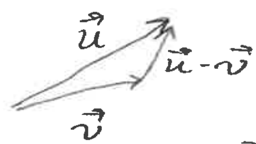


$$\vec{a} = \langle a_1, a_2, a_3 \rangle \quad \text{and} \quad \vec{b} = \langle b_1, b_2, b_3 \rangle$$

$$\vec{a} - \vec{b} = \langle a_1, a_2, a_3 \rangle - \langle b_1, b_2, b_3 \rangle$$
$$= \langle a_1, a_2, a_3 \rangle + \langle -b_1, -b_2, -b_3 \rangle$$
$$= \langle a_1 - b_1, a_2 - b_2, a_3 - b_2 \rangle$$



$$\vec{a} - \vec{b} + \vec{b} = \vec{a}$$

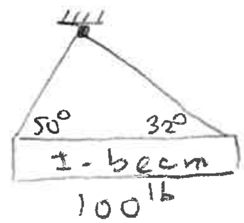


$$\vec{v} + \vec{u} - \vec{v} = \vec{u}$$

Vector Space V $\quad \vec{a}, \vec{b} \in V \quad c, d \in \mathbb{R}$

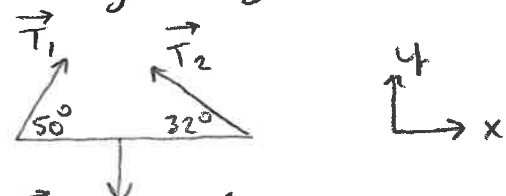
1. $\vec{a} + \vec{b} \in V$ closed under vector addition
2. $\vec{a} + \vec{b} = \vec{b} + \vec{a}$ commutative
3. $(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c})$ associative
4. $\vec{a} + \vec{0} = \vec{a}$ additive identity
5. $\vec{a} + (-\vec{a}) = \vec{0}$ inverse vector
6. $c\vec{a} \in V$ closed under scalar multiplication
7. $c(\vec{a} + \vec{b})$ scalar multiplication distributes over vector addition
8. $(c+d)\vec{a}$ scalar multiplication distributes over scalar addition
9. $(cd)\vec{a} = c(d\vec{a})$ association
10. $1\vec{a} = \vec{a}$ multiplicative identity

Application:



What is the tension in each cable?

Free body diagram



$$\vec{W} = -100\,\hat{j}$$

$$\vec{T}_1 = |\vec{T}_1| \cos 50^\circ \,\hat{i} + |\vec{T}_1| \sin 50^\circ \,\hat{j}$$

$$\vec{T}_2 = -|\vec{T}_2| \cos 32^\circ \,\hat{i} + |\vec{T}_2| \sin 32^\circ \,\hat{j}$$

$$\Sigma \vec{F}_x = \vec{0} \qquad |\vec{T}_1| \cos 50^\circ - |\vec{T}_2| \cos 32^\circ = 0$$

$$\Sigma \vec{F}_y = \vec{0} \qquad |\vec{T}_1| \sin 50^\circ + |\vec{T}_2| \sin 32^\circ - 100 = 0$$

Rearrange the first equation to get $|\vec{T}_2| = |\vec{T}_1| \dfrac{\cos 50^\circ}{\cos 32^\circ}$

Substitute the result into 2nd eq. to get

$$|\vec{T}_1| \sin 50^\circ + |\vec{T}_1| \frac{\cos 50^\circ}{\cos 32^\circ} \sin 32^\circ = 100 \Rightarrow$$

$$|\vec{T}_1| = \frac{100}{\sin 50^\circ + \cos 50^\circ \tan 32^\circ} \approx 85.64^{lb}$$

$$|\vec{T}_2| \approx 85.64 \frac{\cos 50^\circ}{\cos 32^\circ} \approx 64.91^{lb}$$

§2.3 Dot Product

Let $\vec{a}, \vec{b} \in V$. The dot product (inner product or scalar product) is defined as

$$\vec{a} \cdot \vec{b} = \langle a_1, a_2, a_3 \rangle \cdot \langle b_1, b_2, b_3 \rangle$$
$$= a_1 b_1 + a_2 b_2 + a_3 b_3$$

$$\vec{a} = 2\hat{i} - 4\hat{j} + 5\hat{k}$$
$$\vec{b} = 3\hat{i} + 2\hat{j} - 6\hat{k}$$

$$\vec{a} \cdot \vec{b} = 2(3) - 4(2) + 5(-6) = 6 - 8 - 30 = -32$$

Example: We sell hot dogs for \$3, chips for \$2, and water for \$1. We sold 30 hot dogs, 20 chips, and 25 water bottles. What is our revenue.

$$R = \vec{P} \cdot \vec{C} = \langle 30, 20, 25 \rangle \cdot \langle 3, 2, 1 \rangle$$
$$= 90 + 40 + 25 = \$155$$

Example: $\vec{a} = \langle 5, 7 \rangle$

$\vec{b} = \langle 7, -5 \rangle$

$$\vec{a} \cdot \vec{b} = 35 - 35 = 0$$

Two vectors are orthogonal (perpendicular) if and only if their dot product is 0.