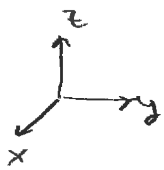
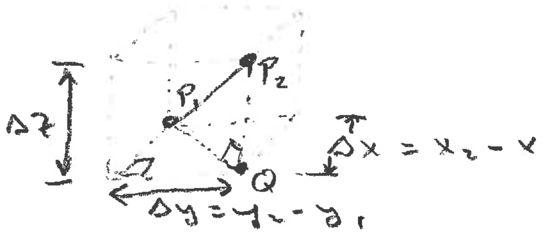
Tues 8/19

§12.1 3D Coordinate System

Consider $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$.





We want to find $|P_1P_2|$.

Then $|P_1Q| = \sqrt{(\Delta x)^2 + (\Delta y)^2}$ and $|QP_2| = \Delta z$

$$|P_1P_2| = \sqrt{|P_1Q|^2 + |QP_2|^2} = \sqrt{\sqrt{(\Delta x)^2 + (\Delta y)^2}^{\,2} + (\Delta z)^2}$$

$$= \sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2}$$

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Let $P_1(7, -3, 5)$ and $P_2(4, 1, -7)$.

Find the distance $|P_1P_2|$.

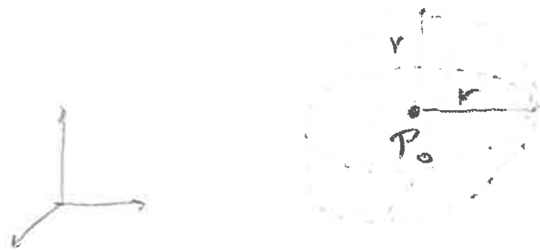
$$|P_1P_2| = \sqrt{(4-7)^2 + (1+3)^2 + (-7-5)^2}$$

$$= \sqrt{3^2 + 4^2 + 12^2} = \sqrt{9 + 16 + 144} = \sqrt{169} = 13$$

Describe the following set:

$$\{ P(x,y,z) \mid |PP_0| = r \}$$

where $P_0(x_0, y_0, z_0)$ and $r > 0$.



This set represents a sphere. Therefore, the eq. of a sphere is determined by $|PP_0|^2 = r^2$ which is

$$(x-x_0)^2 + (y-y_0)^2 + (z-z_0)^2 = r^2$$

Note this has constant, linear, and quadratic terms. This is called a quadric surface.

Contrast this with the eq. of a plane

$$a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$$

this has only linear and constant terms.

Example: Write the equation of a sphere for center $(-4, 2, 7)$ and radius 7.

$$(x-(-4))^2 + (y-2)^2 + (z-7)^2 = 7^2$$

$$(x+4)^2 + (y-2)^2 + (z-7)^2 = 49$$

Example: Show that the following eq. is an eq. of a sphere.

$$x^2 + y^2 + z^2 + 4x - 6y - 2z = 22$$

$$x^2 + 4x + 4 + y^2 - 6y + 9 + z^2 - 2z + 1 = 22 + 4 + 9 + 1$$

$$(x+2)^2 + (y-3)^2 + (z-1)^2 = 36$$

center is $(-2, 3, 1)$

radius is 6

Example: Find an eq of a sphere with points $P(x,y,z)$ such that the distance from P to $A(-3,6,2)$ is twice the distance from P to $B(6,2,-1)$.

$$|PA| = 2|PB| \Rightarrow |PA|^2 = 4|PB|^2 \Rightarrow$$

$$(x+3)^2 + (y-6)^2 + (z-2)^2 = 4[(x-6)^2 + (y-2)^2 + (z+1)^2]$$

$$x^2 + 6x + 9 + y^2 - 12y + 36 + z^2 - 4z + 4$$

$$= 4(x^2 - 12x + 36 + y^2 - 4y + 4 + z^2 + 2z + 1)$$

$$= 4x^2 - 48x + 144 + 4y^2 - 16y + 16 + 4z^2 + 8z + 4 \Rightarrow$$

$$3x^2 - 54x + 3y^2 - 4y + 3z^2 + 12z = 9 + 36 + 4 - 144 - 16 - 4$$

$$= -115 \Rightarrow$$

$$x^2 - 18x + 81 + y^2 - \frac{4}{3}y + \frac{4}{9} + z^2 + 4z + 4 = -\frac{115}{3} + 81 + \frac{4}{9} + 4$$

$$(x-9)^2 + (y - \tfrac{2}{3})^2 + (z+2)^2 = \frac{424}{9}$$

center is $(9, \frac{2}{3}, -2)$

radius is $\frac{\sqrt{424}}{3}$