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math 142
9/23/22

7.1 Integration by parts

How to integrate a product $\int u(x)v(x)\,dx$

Consider the product rule:

$$\int \frac{d}{dx} u(x)v(x)\,dx = \int (u'(x)v(x) + u(x)v'(x))\,dx$$

$$u(x)v(x) = \int u'(x)v(x)\,dx + \int u(x)v'(x)\,dx$$

$$\int u(x)v'(x)\,dx = u(x)v(x) - \int u'(x)v(x)\,dx$$

$$\int u\,dv = uv - \int v\,du$$

Integration by parts formula

- need dv integrable
- want u simplifed by differentiation

ex: $\int x \cdot e^x dx$

$u = x \quad dv = e^x dx$

$du = 1dx \quad v = e^x$

$uv - \int v du$

$= x \cdot e^x - \int e^x dx$

$= xe^x - e^x + c$

$= \boxed{e^x(x-1)+c}$

ex: $\int \ln x \, dx$

$uv - \int v du$

$u = \ln x \quad dv = dx$

$du = \frac{1}{x} dx \quad v = x$

$= (\ln x)x - \int x \frac{1}{x} dx$

$= (\ln x)x - \int 1 dx$

$\boxed{= (\ln x)x - x + c}$

$= x(\ln x - 1) + c$

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7.1 (con.)

ex: $\int \arctan x \, dx$

$u = \arctan x \qquad dv = dx$

$du = \frac{1}{1+x^2} dx \qquad v = x$

$= (\arctan x)x - \int x \cdot \frac{1}{1+x^2} dx$

$r = 1+x^2 \quad dr = 2x\,dx$

$\frac{1}{2}\int \frac{dr}{r} = \frac{1}{2}\ln|r| = \frac{1}{2}\ln(1+x^2)$

$$\boxed{= x \arctan x - \frac{1}{2}\ln(1+x^2) + C}$$

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7.1

$\int u \cdot dv = u \cdot v - \int v\,du$

LIATE

$\leftarrow u \quad dv \rightarrow$

- guide to choose u, dv

Logarithm
Inverse trig
Algebra
Trig
Exponential

$\uparrow u \quad dv \downarrow$

ex $\int_1^2 x \ln x \, dx$

$u = \ln x \qquad dv = x\,dx$

$du = \frac{1}{x}\,dx \qquad v = \frac{1}{2}x^2$

$= \ln x \cdot \frac{1}{2}x^2 \Big|_1^2 - \int_1^2 \frac{1}{2}x^2 \frac{1}{x}\,dx$

$= \frac{1}{2}x^2 \ln x \Big|_1^2 - \int_1^2 \frac{1}{2}x\,dx$

$= \left(\frac{1}{2}x \ln x - \frac{1}{4}x^2\right)\Big|_1^2 = (2\ln 2 - 1) - \left(0 - \frac{1}{4}\right)$

$= \boxed{2\ln 2 - \frac{3}{4}}$

$\int u \cdot dv = u \cdot v - \int v\, du$

ex: $\int x^3 \sin x\, dx$

$u = x^3$ $\quad dv = \sin x\, dx$ $\quad = x^3(-\cos x) - \int -\cos x(3x^2)\,dx$

$du = 3x^2 dx$ $\quad v = -\cos x$

$= -x^3\cos x + 3\int x^2 \cos x\, dx$

$u = x^2$ $\quad dv = \cos x\, dx$ $\quad = [-x^3\cos x + 3] 3[x^2 \sin x - \int \sin \cdot 2x\, dx]$

$du = 2x\,dx$ $\quad v = \sin x$

$-2[\int x \sin x\, dx]$

$u = x$ $\quad dv = \sin x\, dx$ $\quad x(-\cos x) - \int -\cos x\, dx$

$du = dx$ $\quad v = -\cos x$ $\quad + \sin x$

$= \boxed{-x^3\cos x + 3x^2\sin x + 6x\cos x - 6\sin x + C}$

OI method

Stop when row product is integrable or repeated.

O		I
x^3	+	$\sin x$
$3x^2$	−	$-\cos x$
$6x$	+	$-\sin x$
6	−	$\cos x$
0		$\sin x$

$-x^3\cos x + 3x^2\sin x + 6x\cos x - 6\sin x + C$

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7.1

$\int (\ln x)^n dx$, $n \geq 2$

Find a formula in terms of a lower power of $\ln x$

$u = (\ln x)^n$ $dv = dx$

$du = n(\ln x)^{n-1} \cdot \frac{1}{x} dx$ $v = x$

$= (\ln x)^n x - \int \cancel{x} n (\ln x)^{n-1} \cdot \frac{1}{\cancel{x}} dx$

$= \boxed{x(\ln x)^n - n\int (\ln x)^{n-1} dx}$

Reduction formula

$n=2$

$\int (\ln x)^2 dx = x(\ln x)^2 - 2\int \ln x \, dx$

$= x(\ln x)^2 - 2\int (\ln x)^1 dx$

Find R.F. for $\int \sin^n x \, dx$

$u = (\sin x)^{n-1}$ $dv = \sin x \, dx$

$du = (n-1)\sin^{n-2} x \cdot \cos x \, dx$ $v = -\cos x$

$= (\sin^{n-1} x)(-\cos x) - \int -\cos x (n-1) \sin^{n-2} x \cdot \cos x \, dx$

$= -\cos x \cdot \sin^{n-1} x \quad + (n-1) \int \cos^2 x \cdot \sin^{n-2} x \, dx$

$(1-\sin^2 x)\sin^{n-2} x$

$\int \sin^n x \, dx = -\cos x \sin^{n-1} x + \int \sin^{n-2} x \, dx - \underline{(n-1)} \int \sin^n x \, dx$

$n \int \sin^n x \, dx = -\cos x \sin^{n-1} x + (n-1) \int \sin^n x \, dx$

$\int \sin^n dx = \boxed{\frac{-1}{n} \cos x \sin^{n-1} x + \frac{n-1}{n} \int \sin^{n-2} x \, dx}$

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7.1

$\int x^5 \cdot e^{x^3} dX$ $\quad r = x^3$

$x^2 \cdot x^3$ $\quad dr = 3x^2 dX$

$\frac{1}{3}\int r \cdot e^r dr$ $\quad u = r \quad dv = e^r dr$

$du = dr \quad v = e^r$

$= \frac{1}{3} re^r - \int e^r dr$

$= \frac{1}{3} re^r - e^r + C$

$= \frac{1}{3} e^{x^3} (x^3 - 1) + C$

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7.1 (Con)

ex: $\int x\sqrt{3x+2}\,dx$

$u = x \qquad dv = \sqrt{3x+2}\,dx$

$du = dx \qquad v = \frac{2}{3}(3x+2)^{3/2} \cdot \frac{1}{3}$ ← u-sub not write

$= \frac{2}{9}(3x+2)^{3/2} \cdot x - \int \frac{2}{9}(3x+2)^{3/2}\,dx$

$= \frac{2}{9}x(3x+2)^{3/2} - \frac{2}{9} \cdot \frac{1}{3} \cdot \frac{2}{5}(3x+2)^{5/2} + C$

$\int x\sqrt{3x+2}\,dx$

$u = 3x+2$

$du = 3\,dx$

$x = \frac{u-2}{3}$

$$= \frac{1}{3}\int \frac{u-2}{3} \cdot \sqrt{u}\,du$$

$$= \frac{1}{9}\int (u^{3/2} - 2u^{\frac{1}{2}})\,du$$

$$= \frac{1}{9}\left(\frac{2}{5}u^{5/2} - \frac{4}{3}u^{3/2}\right) + C$$

$$= \frac{2}{45}(3x+2)^{5/2} - \frac{4}{27}(3x+2)^{3/2} + C$$

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7.1 con

ex: $\int \sin(\ln x)\,dx$

$r = \ln x$

$dr = \frac{1}{x}\,dx$

$dx = x\,dr$

$= e^r\,dr$

$= \int e^r \cdot \sin r\,dr$

	D		I
	$\sin r$	$+$	e^r
	$\cos r$	$-$	e^r
$e^r \sin r - e^r \cos r - \int e^r \sin r\,dr$	$-\sin r$		e^r

$2\int e^r \sin r\,dr = e^r(\sin r - \cos r)$

$\int e^r \sin r\,dr = \frac{e^r}{2}(\sin r - \cos r) + C$

$\int \sin(\ln x)\,dx = \boxed{\frac{x}{2}(\sin(\ln x) - \cos(\ln x)) + C}$