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Math 142
4/20/22

6.4 work

$W = F \cdot d$
work = force · distance
(displacement)

units

US foot-pound
SI Newton-meters = Joule

example: crane lifts 3000lb car 50ft

$W = (3000\,lb)(50\,ft) = \boxed{150{,}000\,ft\text{-}lb}$

variable force $W \approx \sum f(x_i)\,\Delta x$

take limit $W = \int_a^b f(x)\,dx$

ex: Gravity $F = \frac{GMm}{r^2}$ work to escape Earth.

$$W = \int_{r_0}^{\infty} -\frac{GMm}{r^2}\,dr = -GMm \int_{r_0}^{\infty} r^{-2}\,dr$$

$$= +\frac{GMm}{r}\Big|_{r_0}^{\infty} = 0 + \frac{GMm}{r_0}$$

finite!

Hooke's law (springs)

force to stretch spring X units from natural length

$F = kx$ k - spring constant

ex: spring - natur. length 2ft
- 18 lb stretches to 3.2 ft

work to stretch from 2ft to 4ft?

$F = kx$ $\quad x = 3.2 - 2 = 1.2$

$18 = k \cdot 1.2$
$k = 15$

$$W = \int_0^2 15x\,dx$$

$$= \frac{15x^2}{2}\Big|_0^2 = \frac{15(2)^2}{2} - 0 = \boxed{30 \text{ ft-lbs}}$$

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9/20/22

6.4 (Con.)

ex. work 48 J to stretch from n.l. 8m to 12m. work to stretch from 12 to 15 m?

$$W = \int_0^4 kx\,dx = \frac{1}{2}kx^2\Big|_0^4 = 8k$$

$$48 = 8k \rightarrow k = 6$$

$$W = \int_4^7 6x\,dx = 3x^2\Big|_4^7 = 3(49-16) = \boxed{99\text{ J}}$$

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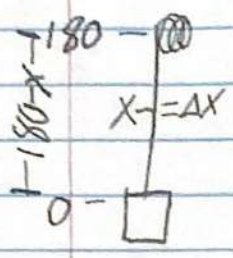
6.4

ex: An elevator cable has density 4.5 lb/ft, length 180 ft. How much work to lift all to same height?



$$\Delta W = 4.5(180-x)\Delta x$$

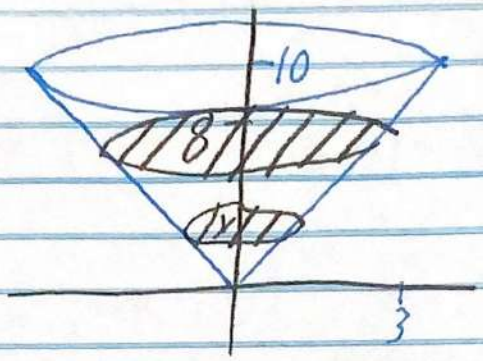
$$W = \int_0^{180} 4.5(180-x)\,dx$$

$$= 4.5\int_0^{180}(180-x)\,dx$$

$$= 4.5\left(180x - \frac{x^2}{2}\right)\Big|_0^{180}$$

$$= \frac{9}{2}\cdot\frac{1}{2}\,180^2 = 9\cdot 90^2 = \boxed{72900 \text{ ft-lbs}}$$

ex. A conical tank filled within 2ft of top with oil weighing 57 lb/ft^3. How much work to pump to top?



$$\Delta V = \pi r^2 \Delta y$$

$$= \pi \left(\frac{y}{2}\right)^2 \Delta y$$

$$= \frac{\pi}{4} y^2 \Delta y$$

$$\frac{r}{y} = \frac{5}{10} \qquad r = \frac{y}{2}$$

$$\Delta F = 57 \cdot \Delta V = \frac{57}{4} \pi y^2 \Delta y - lb$$

$$\Delta W = \Delta F \cdot (10 - y) - ft\,lbs$$

$$W = \int_0^8 \frac{57}{4} \pi y^2 (10 - y)\, dy$$

$$\frac{57}{4} \pi \int_0^8 (10y^2 - y^3)\, dy$$

$$\frac{57}{4} \pi \left[\frac{10y^3}{3} - \frac{y^4}{4} \right] \Bigg|_0^8 \approx \boxed{30,561\ ft\,lbs}$$

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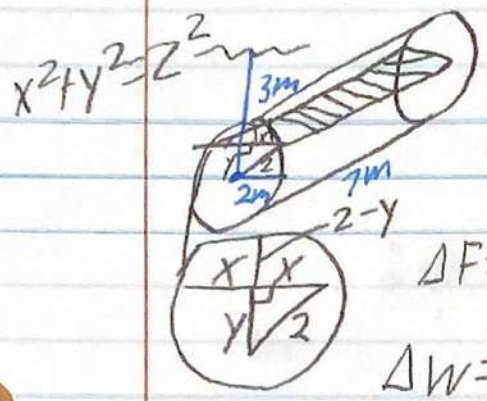
6.4

ex: A tank is a cylinder on its side full of gas ($\delta = 673\ kg/m^3$) 3m underground. Work to pump to surface?



$$\Delta V = 7(2x)\Delta y$$

$$= 14\sqrt{4-y^2}\,\Delta y \quad m^3 \qquad F = mg$$

$$\Delta F = 673\,\Delta V \cdot 9.8 \qquad N = kg \cdot \frac{m}{s^2}$$

$$\Delta W = \Delta F(5-y) \qquad Nm$$

$$W = \int_{-2}^{2} 673 \cdot 9.8 \cdot 14(5-y)\sqrt{4-y^2}\,dy$$

$$\approx \boxed{2,900,808 \quad J}$$