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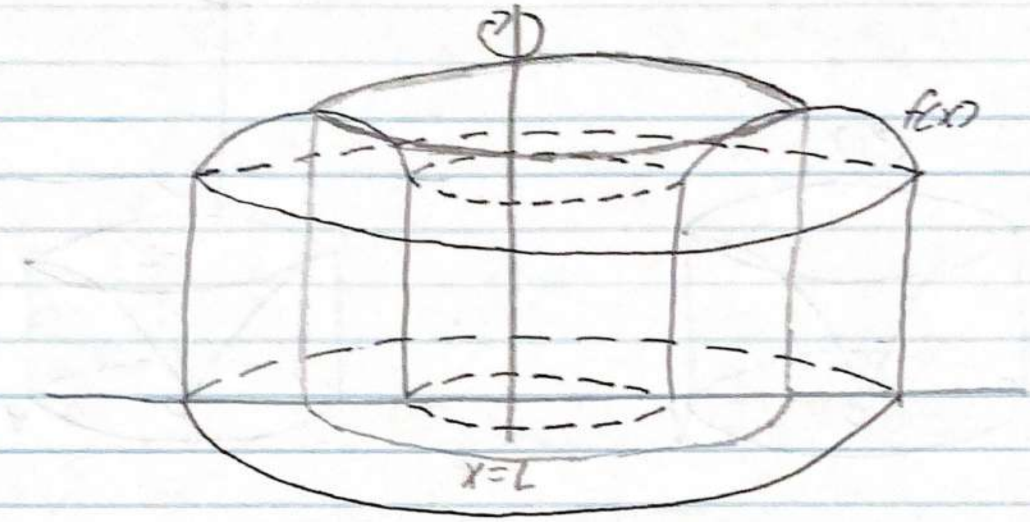
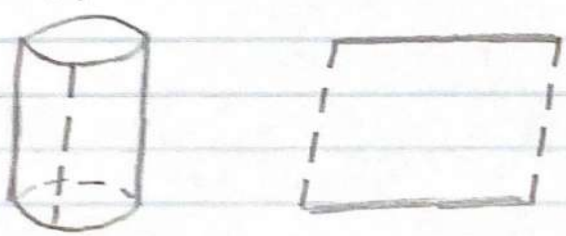
Math 142
9/19/22

6.3 the shell method



- Slice solid into cylindical shells
- Approx. Shells with thin boxes



$\Delta V_k \approx$ circumference • height • Thickness

$$\approx 2\pi(y_k - L) f(y_k) \Delta y_k$$

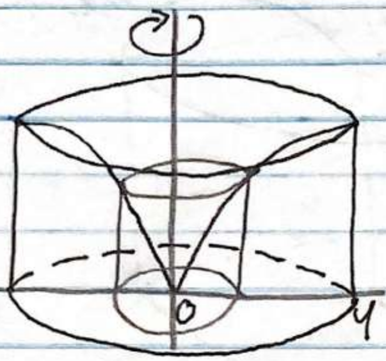
$V \approx \sum V_k$ take limit

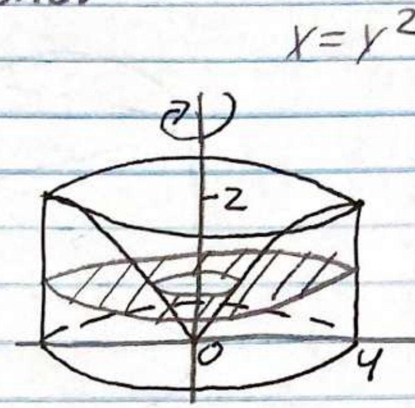
$$V = \int_a^b 2\pi(x-L) f(x)\, dx$$

The shell method

ex: $f(x) = \sqrt{x}$, $[0, 4]$, y-Axis

- use both shell and washer





The shell method

$$V = \int_0^4 2\pi x\sqrt{x}\,dx$$

$$= 2\pi \int_0^4 x^{3/2}\,dx$$

$$= 2\pi \frac{2}{5} x^{5/2} \Big|_0^4 = \boxed{\frac{128}{5}\pi}$$

the washer method

$$V = \pi \int_0^2 \left(4^2 - (y^2)^2\right) dy$$

$$= \pi \int_0^2 16 - y^4\,dy$$

$$= \pi\ 16y - \frac{1}{5}y^5 \Big|_0^2 = \pi\left(32 - \frac{32}{5}\right) = \boxed{\frac{128}{5}\pi}$$

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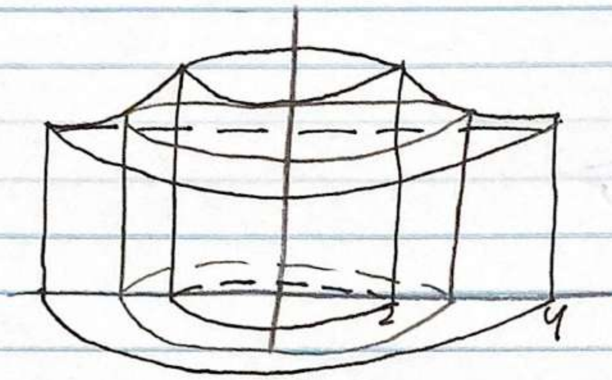
Math 142
9/13/22

6.3 (con.)

ex: $f(x) = \frac{1}{x^2}$, $[2,4]$, y-Axis

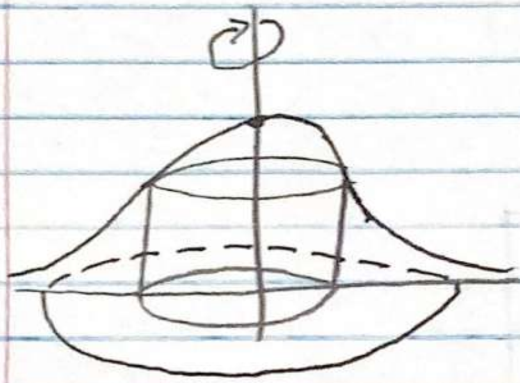


$$V = \int_2^4 2\pi x \frac{1}{x^2}\,dx = 2\pi \int_2^4 \frac{1}{x}\,dx$$

$$= 2\pi \ln|x| \Big|_2^4 = 2\pi(\ln 4 - \ln 2) = 2\pi \ln \frac{4}{2}$$

$$= \boxed{2\pi \ln 2}$$

ex: $f(x)=e^{-x^2}$, $[0,\infty)$, y-Axis



$$V=\int_0^\infty 2\pi x e^{-x^2}\,dx$$

$u=-x^2$
$du=-2x\,dx$

$$=-\pi\int_{u=0}^{u=-\infty} e^u\,du=-\pi e^u\Big|_0^{-\infty}$$

$$=-\pi(0-1)=\pi$$

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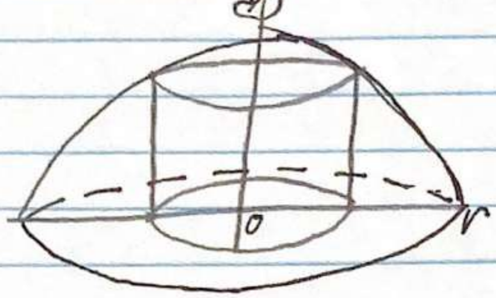
Math 142
9/13/22

6.3 (con.)

ex: Sphere $y=\sqrt{r^2-x^2}$, y-Axis



$V = 2\int_0^r 2\pi x\sqrt{r^2-x^2}\,dx$

$u = r^2-x^2$

$du = -2x\,dx$

$= 2\pi \int_{u=r^2}^{u=0} \sqrt{u}\,du$

$= 2\pi \left(\frac{2}{3}u^{3/2}\right)\Big|_{r^2}^{0}$

V_{sp} $\boxed{= \frac{4}{3}\pi r^3}$

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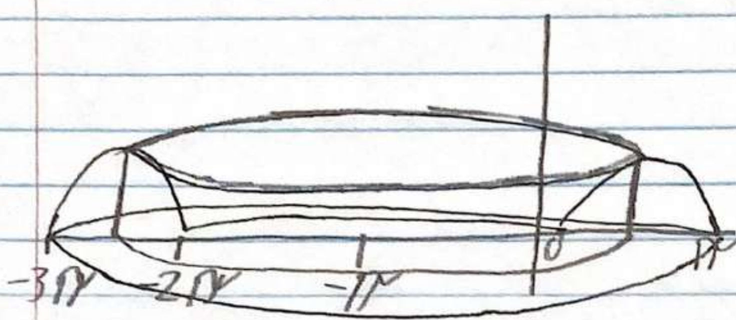
Math 142
09/20/22

6.3 the shell method (con.)

$$V=\int_a^b 2\pi(x-L)f(x)\,dx$$

ex: $y-\sin x$, $[0,\pi]$ about $x=-\pi$



$$V=\int_0^{\pi} 2\pi(x+\pi)\sin x\,dx$$