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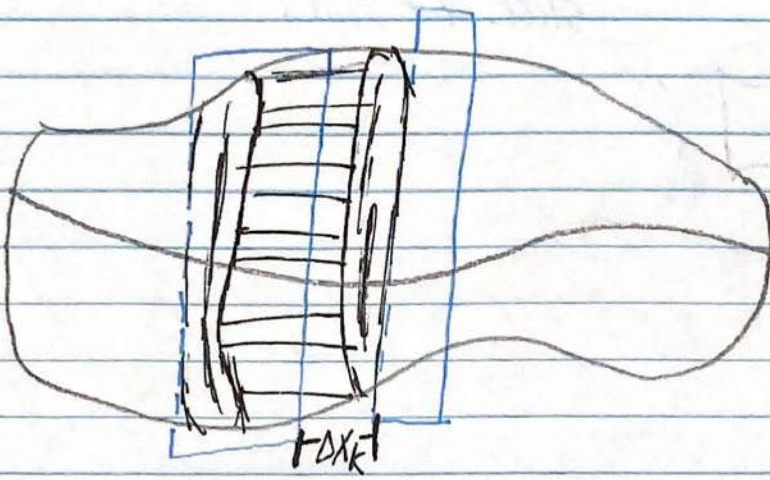
Math 142
9/14/22

6.2 volumes

- Slice solid into cross-sections
- Approximate slices with "cylinders"



$V(k^{th} \text{ slice}) \approx V(k^{th} \text{ cylinder})$

$= A(x_k) \cdot \Delta x_k$

$V \approx \Sigma V_k$

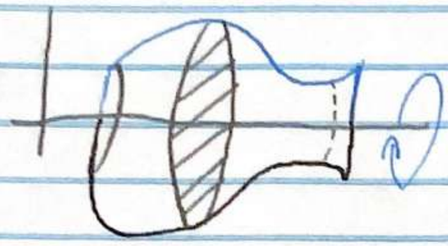
$= \Sigma A(x_k) \Delta x_k$ Riemann Sum

take limit $\rightarrow \int_a^b A(x)\, dx$

volume of solid with cross-sectional area $A(x)$ is $V = \int_a^b A(x)\,dx$

Solid of revolution
- rotate plan region around axis

slices are disks



$A(x) = \pi r^2 = \pi [R(x)]^2$

$$\boxed{V = \pi \int_a^b [R(x)]^2\,dx}$$

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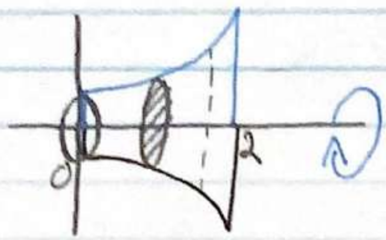
Math 142
9/13/22

6.2 (con.)

ex. $R(x) = e^x$, $[0,2]$ about x-Axis



$$V = \pi \int_0^2 (e^x)^2 \, dx$$

$$= \pi \int_0^2 e^{2x} \, dx$$

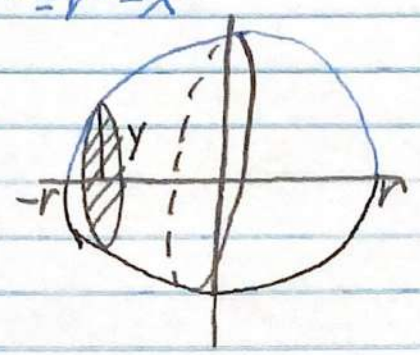
$$= \pi \cdot \frac{1}{2} e^{2x} \Big|_0^2$$

$$= \boxed{\frac{\pi}{2}(e^4 - 1)}$$

Sphere - rotate circle

$x^2 + y^2 = r^2$ about x-axis

$y^2 = r^2 - x^2$



$$V = \pi \int_{-r}^{r} (r^2 - x^2)\, dx$$

$r^2 = C$

$$= \pi \left(r^2 x - \frac{x^3}{3} \right) \Big|_{-r}^{r}$$

$$= \left(r^3 - \frac{1}{3} r^3 - \left(-r^3 + \frac{1}{3} r^3 \right) \right)$$

$$\boxed{V_{sp} = \frac{4}{3} \pi r^3}$$

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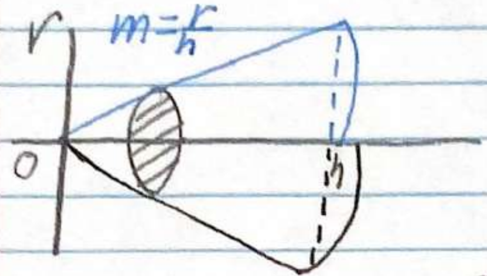
Math 142
9/13/22

6.2 (con.)

Cone – base radius r
height h



$$V=\pi\int_0^h \left(\frac{r}{h}x\right)^2 dx$$

$$=\pi\frac{r^2}{h^2}\int_0^h x^2 dx$$

$$=\pi\frac{r^2}{h^2}\,\frac{1}{3}x^3\Big|_0^h=\pi\frac{r^2}{h^2}\cdot\frac{1}{3}h^3-0$$

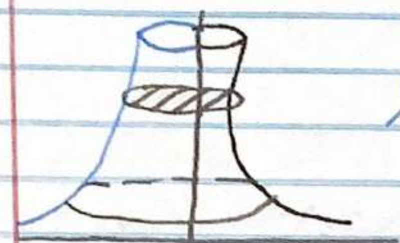
$$\boxed{V_{cone}=\frac{1}{3}\pi r^2 h}$$

Revolve around y-axis

$V = \int_a^b A(y)\,dy$

ex. $y = \frac{1}{x^2}$ about y-axis

$1 \le y \le 3$



$x = \frac{1}{\sqrt{y}}$

$$V = \pi \int_1^3 \left(\frac{1}{\sqrt{y}}\right)^2 dy$$
$$= \pi \int_1^3 \frac{1}{y}\,dy$$
$$= \pi \ln|y| \Big|_1^3$$
$$= \pi \ln 3$$

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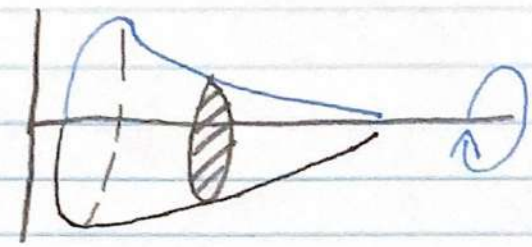
Math 142
9/14/22

6.2 (con.)

Gabriel's Trumpet

$y=\frac{1}{x}$, $[1,\infty)$ about x-axis



$V=\pi\int_1^\infty \left(\frac{1}{x}\right)^2 dx = \pi\int_1^\infty x^{-2}dx$

$=\pi\left(-\frac{1}{x}\right)\Big|_1^\infty = \pi(0--1)=\pi$

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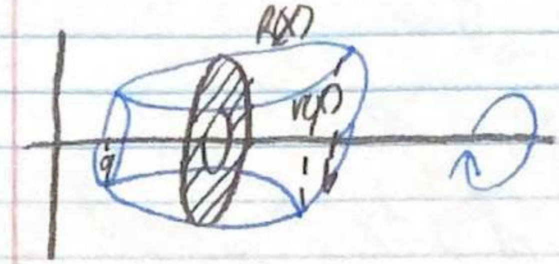
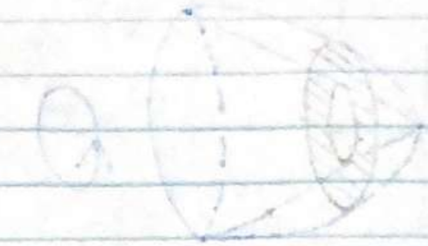
Math 142
09/16/22

6.2 (con.) Region between two curves

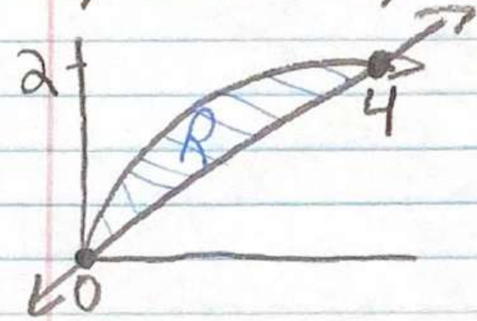
- Slices are





$$V = \pi \int_a^b \left(R^2(x) - r^2(x)\right) dx$$ washer method

$y = \sqrt{x}$ and $y = \frac{1}{2}x$



$$\frac{1}{2}x = \sqrt{x}$$

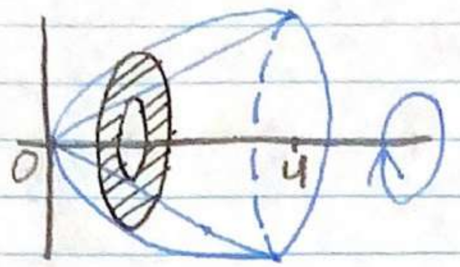
$$\frac{1}{4}x^2 = x$$

$$\frac{1}{4}x^2 - x = 0$$

$$x\left(\tfrac{1}{4}x - 1\right) = 0$$

$$x = 0, 4$$

R about x-axis



$$V = \pi \int_0^4 \left(\sqrt{x}^{\,2} - \left(\tfrac{1}{2}x\right)^2\right) dx$$

$$= \pi \int_0^4 \left(x - \tfrac{1}{4}x^2\right) dx$$

$$= \pi \left(\tfrac{1}{2}x^2 - \tfrac{1}{12}x^3\right)\Big|_0^4$$

$$= \pi\left(8 - \tfrac{16}{3}\right) = \boxed{\tfrac{8}{3}\pi}$$

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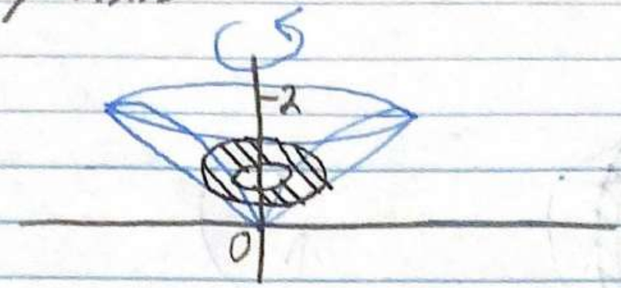
Math142
9/16/22

R about y-Axis



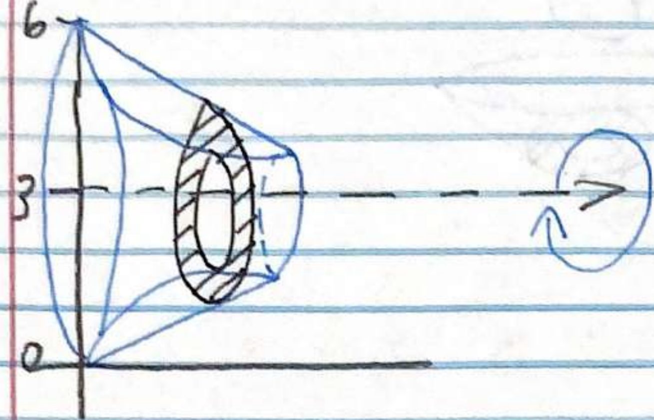
$$V=\pi\int_0^2\left((2y)^2-(y^2)^2\right)dy$$

$$=\pi\int_0^2(4y^2-y^4)\,dy$$

$$=\pi\left(\frac{4}{3}y^3-\frac{1}{5}y^5\right)\Big|_0^2$$

$$=\pi\left(\frac{32}{3}-\frac{32}{5}\right)=\boxed{\frac{64}{15}\pi}$$

R about $y=3$



$$V=\pi\int_0^4\left(\left(3-\tfrac{1}{2}x\right)^2-\left(3-\sqrt{x}\right)^2\right)dx$$

$$=\pi\int_0^4\left(9-3x+\tfrac{1}{4}x^2-(9-6\sqrt{x}+x\right)dx$$

$$=\pi\int_0^4\left(\tfrac{1}{4}x^2-4x+6\sqrt{x}\right)dx$$

$$=\pi\left(\tfrac{1}{12}x^3-2x^2+4x^{3/2}\right)\Big|_0^4$$

$$=\boxed{\tfrac{16}{3}\pi}$$

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Math 142
9/16/22

General Solids
1. Sketch solid, cross-section
2. Find formula for $A(x)$
3. Find limits of integration
4. Integrate $A(x)$.

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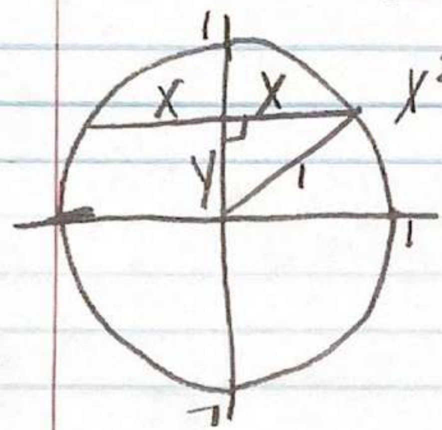
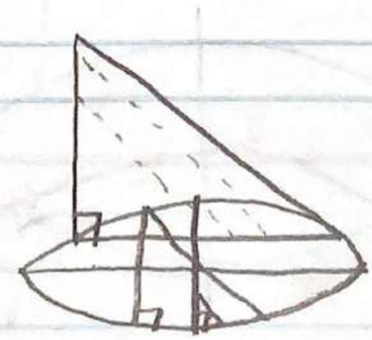
ex solid with unit circle base, slices are isosceles right triangles



$x^2+y^2=1$



$$A(y)=\frac{1}{2}bh=\frac{1}{2}(2x)^2=2x^2=2(1-y^2)$$

$$V=\int_{-1}^{1}2(1-y^2)\,dy$$

$$2\left(y-\frac{y^3}{3}\right)\Big|_{-1}^{1}$$

$$2\left[\left((1)-\frac{(1)^3}{3}\right)-\left((-1)-\frac{(-1)^3}{3}\right)\right]=2\left(\frac{2}{3}--\frac{2}{3}\right)$$

$$=\boxed{\frac{8}{3}}$$

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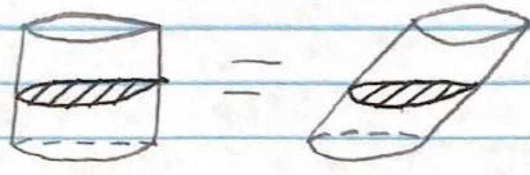
math 142
9/19/22

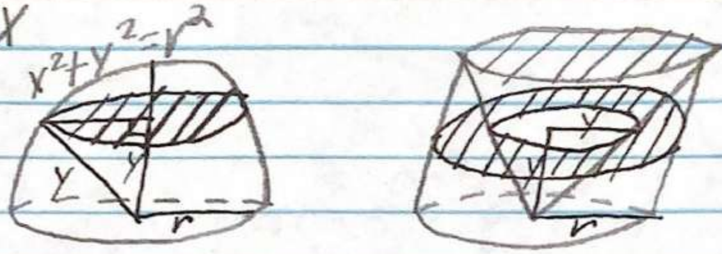
6.2 (con.)

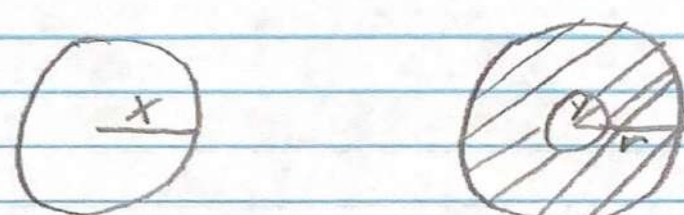
$V=\int_a^b A(x)\,dx$

cavalieri's principle: slices have same area
same volume



ex





$A_{hs}(y)=\pi x^2=\pi(r^2-y^2)\overset{!}{=}\pi r^2-\pi y^2=A_{cc}(y)$

$V_{cc}=\pi r^2 r-\frac{1}{3}\pi r^2 r=\frac{2}{3}\pi r^3$

$V_{sp}=2V_{hs}=2V_{cc}=\frac{4}{3}\pi r^3$