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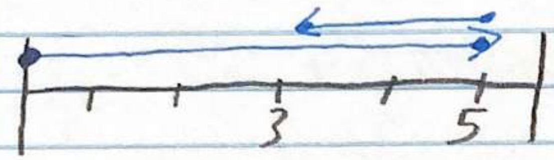
9/12/22
Math 142

# 6.1 Area Between Curves

Displacement - distance between start and end points

distance traveled - total distance

example:

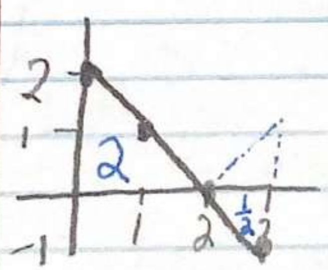



$disp = 5 - 2 = 3$     $disp \leq D.T.$

$DT = 5 + 2 = 7$

ex: $v(t) = 2 - t$, $[0, 3]$

Find disp., DT




$disp = 2 - \frac{1}{2} = \frac{3}{2}$

$DT = 2 + \frac{1}{2} = \frac{5}{2}$

$disp = \int_0^3 (2-t)\,dt = 2t - \frac{t^2}{2}\Big|_0^3$

$\left[2(3) - \frac{(3)^2}{2}\right] - \left[2(0) - \frac{(0)^2}{2}\right]$

$6 - \frac{9}{2} = \frac{3}{2}$

Def. the total area between $f(x)$ and X-Axis on $[a,b]$ is $\int_a^b |f(x)|\,dx$
(abs)

1. Subdivide $[a,b]$ where $f(x)$ changes sign (zeros).

2. Find the antid. $F(x)$

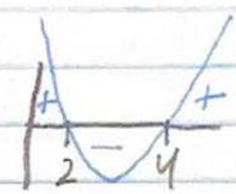
3. Add absolute values of Integrals on subintervals.

ex: $v(t) = 3t^2 - 18t + 24 \quad [0,5]$
find disp, DT

$$s(t) = \frac{3t^3}{3} - \frac{18t^2}{2} + 24t = t^3 - 9t^2 + 24t$$

$$\text{disp} = s(t)\Big|_0^5 = 125 - 225 + 120 = 20$$

$$v(t) = 3(t^2 - 6t + 8) = 3(t-2)(t-4)$$




$$DT = \int_0^2 |v(t)|dt + \int_2^4 |v(t)|dt + \int_4^5 |v(t)|\,dt$$

$$= \int_0^2 v(t)\,dt - \int_2^4 v(t)\,dt + \int_4^5 v(t)\,dt$$

$$= s(t)\Big|_0^2 - s(t)\Big|_2^4 + s(t)\Big|_4^5$$

$$= (s(2) - s(0)) - (s(4) - s(2)) + (s(5) - s(4))$$

$$= s(2) + 2s(2) - 2s(4) + s(5)$$

$$= -0 + 2\cdot 20 - 2\cdot 16 + 20 = 28$$

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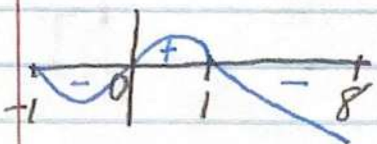
6.1 (con.)

Find TA for $f(x) = x^{\frac{1}{3}} - x$ $[-1, 8]$




$= x^{\frac{1}{3}}(1 - x^{2/3}) = x^{\frac{1}{3}}(1 + x^{\frac{1}{3}})(1 - x^{\frac{1}{3}})$

0 -1 1

$F(x) = \frac{3x^{4/3}}{4} - \frac{x^2}{2}$

$TA = -F(x)\Big|_{-1}^{0} + F(x)\Big|_{0}^{1} - F(x)\Big|_{1}^{8}$

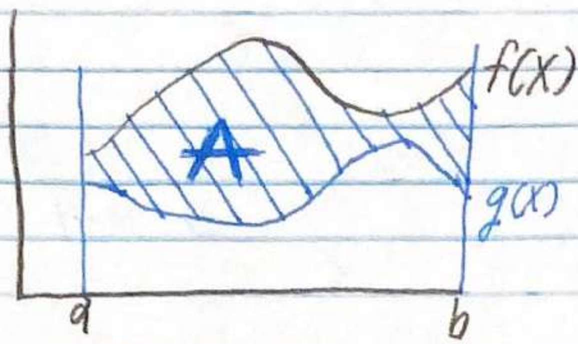
$= -(F(-1) - F(0)) + (F(0) - F(1)) - (F(8) - F(1))$

$= F(-1) - 2F(0) - 2F(1) - F(8)$

$= \frac{1}{4} - 2 \cdot 0 + 2 \cdot \frac{1}{4} - (-20)$

$= \frac{83}{4}$

Area Between curves




$\int_a^b g(x)\,dx + A = \int_a^b f(x)\,dx$

$A = \int_a^b f(x)\,dx - \int_a^b g(x)\,dx$

$= \int_a^b (f(x) - g(x))\,dx \quad (f \geq g)$

$$\boxed{A = \int_a^b \left| f(x) - g(x) \right| dx}$$

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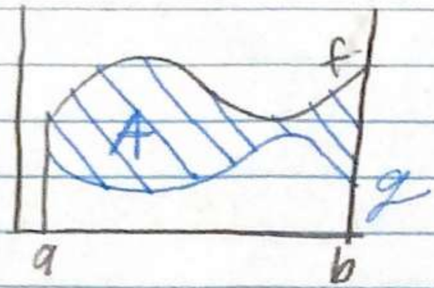
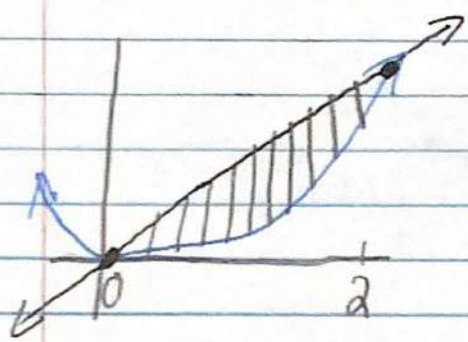
Area between curves




$$A + \int_a^b g(x)\,dx = \int_a^b f(x)\,dx$$

$$A = \int_a^b |f(x) - g(x)|\,dx$$

ex: Find area betwen $Y = X^4$; $Y = 8X$



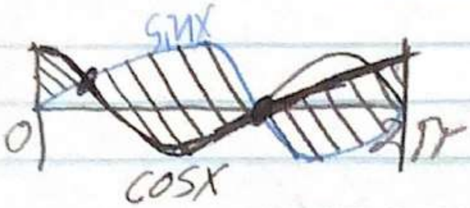

$$X^4 = 8X$$
$$X^4 - 8X = 0$$
$$X(X^3 - 8) = 0$$
$$X = 0, 2$$

$$A = \int_0^2 (8X - X^4)\,dx$$

$$= 4x^2 - \frac{1}{5}x^5 \Big|_0^2$$

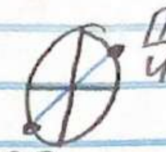
$$= 4(2)^2 - \frac{1}{5}(2)^5 = 16 - \frac{32}{5} - 0 = \boxed{\frac{48}{5}}$$

$y = \sin x, \; y = \cos x \quad [0, 2\pi]$




$\sin x = \cos x$

$\tan x = 1$




$f(x) = \cos x - \sin x$ $\qquad x = \frac{\pi}{4}, \frac{5\pi}{4}$

$F(x) = \sin x + \cos x$

$$A = \int_0^{2\pi} |\cos x - \sin x| \, dx$$

$$+ F(x)\Big|_0^{\frac{\pi}{4}} - F(x)\Big|_{\frac{\pi}{4}}^{\frac{5\pi}{4}} + F(x)\Big|_{\frac{5\pi}{4}}^{2}$$

$$F(\tfrac{\pi}{4}) - F(0) - (F(\tfrac{\pi}{4}) - F(\tfrac{5\pi}{4})) + (F(\tfrac{5\pi}{4}) - F(2))$$

$$= -F(0) + 2F(\tfrac{\pi}{4}) - 2F(\tfrac{5\pi}{4}) + F(2\pi)$$

$$= \cancel{-1} + 2\cdot\sqrt{2} - 2(-\sqrt{2}) \cancel{+1}$$

$$= 4\sqrt{2}$$

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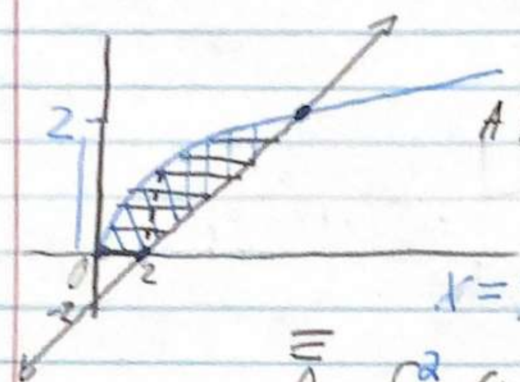
6.1 (con)

$y = \sqrt{x}$, $y = x-2$, x-Axis




$$A = \int_0^2 (\sqrt{x} - 0)\,dx + \int_2 (\sqrt{x} - (x-2))\,dx$$

$x = y^2 = x = y+2$

$y^2 = y+2$

$y^2 - y - 2 = 0$

$(y-2)(y+1) = 0$

$y = 2, -1$

$$A = \int_0^2 (y+2-y^2)\,dy$$

$$= \frac{y^2}{2} + 2y - \frac{y^3}{3} \Big|_0^2$$

$$= \frac{2^2}{2} + 2(2) - \frac{(2)^3}{3} = 2 + 4 - \frac{8}{3} = \frac{10}{3}$$