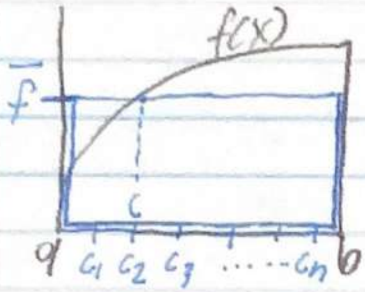
[redacted]

math 142
08/31/22

6.5 Average value of a function



equal area

$$(b-a)\cdot \bar{f} = \int_a^b f(x)\,dx$$

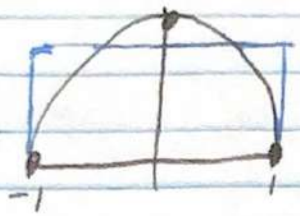
Def. If f(x) is integrable on [a,b], Its average value (AKA mean value) is $\bar{f} = \frac{1}{b-a}\int_a^b f(x)\,dx.$

- Average n evenly spaced function values on [a,b], take limit

$$\frac{1}{n}(f(c_1)+\dots f(c_n)) = \frac{1}{n}\sum_{i=1}^{n} f(c_i)$$

$$= \frac{\Delta x}{b-a}\sum_{i=1}^{n} f(c_i) = \frac{1}{b-a}\sum f(c_i)\Delta X$$

ex. $g(x)=\sqrt{1-x^2}$, $[-1,1]$ find mean value



$$\bar{g} = \frac{1}{1-(-1)}\int_{-1}^{1} g(x)\,dx$$

area of circle

$$= \frac{1}{2}\cdot\frac{\pi}{2} = \frac{\pi}{4} \approx 0.7$$

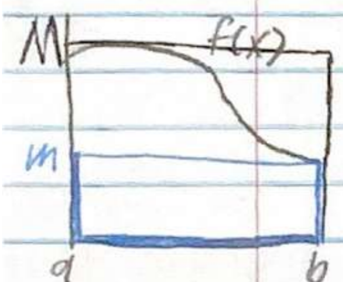
$A=\pi r^2$

Mean value theorem for definite integrals

- If $f(x)$ is continuous on $[a,b]$, there is c in $[a,b]$ with $f(c)=\frac{1}{b-a}\int_a^b f(x)\,dx$

proof:

$$m(b-a) \le \int_a^b f(x)\,dx \le M(b-a)$$



By IVT, f assumes the means at some c in $[a,b]$