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Math 142
09/06/22

5.5 Substitution Rule (u substitution)

- FTC says we need antid. to evaluate integral
- Need techniques to find antid.

recall $\frac{d}{dx} F(g(x)) = f(g(x)) \cdot g'(x)$ chain rule

Then $\int f(\underbrace{g(x)}_{u}) \cdot \underbrace{g'(x)\,dx}_{du} = F(g(x)) + C$ chain rule in reverse

1. Substitute u and du, obtain $\int f(u)\,du$
2. Integrate WRT u. (with respect to)
3. undo the substitution.

ex. $\int 6x\sqrt{3x^2+1}\,dx$

$u = 3x^2+1$ $\qquad = \int \sqrt{u}\,du = \int u^{1/2}\,du$

$du = 6x\,dx$ $\qquad = \frac{2u^{3/2}}{3} + C = \boxed{\frac{2}{3}(3x^2+1)^{3/2} + C}$

$\int x\sin(2x^2)\,dx$

$u = 2x^2$ $\qquad = \int \frac{1}{4}\sin u\,du$

$du = 4x\,dx$

$\frac{1}{4}du = x\,dx$ $\qquad = -\frac{1}{4}\cos u + C = \boxed{-\frac{1}{4}\cos(2x^2) + C}$

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Substitution Rule (con.)

$\int \tan x\,dx = \int \frac{\sin x}{\cos x}\,dx$

$u = \cos x$

$du = -\sin x\,dx$

$= \int \frac{-du}{u} = -\ln|u| + C$

$= \boxed{-\ln|\cos x| + C}$

$= \ln|(\cos x)^{-1}| + C = \boxed{\ln|\sec x| + C}$

$\int \sec x\,dx = \int \sec x \cdot \frac{\sec x + \tan x}{\sec x + \tan x}\,dx$

$= \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x}\,dx$

$u = \sec x + \tan x$

$du = \sec x \tan x + \sec^2 x\,dx$

$= \int \frac{du}{u} = \ln|u| + C$

$\star = \boxed{\ln|\sec x + \tan x| + C}$

$$\int \frac{x}{x-3}\,dx$$

$v = x-3$

$$= \int \frac{v+3}{v}\,dv = \int \left(1+\frac{3}{v}\right)dv$$

$dv = dx$

$\Rightarrow x = v+3$

resubstitution

$$= v + 3\ln|v| + C = \boxed{(x-3)+3\ln|x-3|+C}$$

$$= x + 3\ln|x-3| + C', \quad C' = C-3$$

$$\int x^3 \cdot \sqrt{x^2+1}\,dx$$

$$= \tfrac{1}{2}\int (v-1)\sqrt{v}\,dv$$

$v = x^2+1 \Rightarrow x^2 = v-1$

$dv = 2x\,dx \Rightarrow \frac{1}{2}\,dv = x\,dx$

$$= \frac{1}{2}\int (v^{3/2} - v^{1/2})\,dv$$

$$= \frac{1}{2}\left(\frac{2v^{5/2}}{5} - \frac{2v^{3/2}}{3}\right) + C$$

$$\boxed{= \frac{(x^2+1)^{5/2}}{5} - \frac{(x^2+1)^{3/2}}{3} + C}$$

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5.5 (con)

$$\int \frac{x}{\sqrt{x+1}}\,dx$$

$u = x+1 \Rightarrow u-1 = x$

$du = dx$

$= \int \frac{u-1}{\sqrt{u}}\,du$

$= \int \frac{u}{\sqrt{u}} - \int \frac{1}{\sqrt{u}}\,du$

$= \frac{2}{3}u^{3/2} - 2u^{1/2} + C$

$\boxed{= \frac{2(x+1)^{3/2}}{3} - 2(x+1)^{1/2} + C}$

$u = \sqrt{x+1} \Rightarrow x = u^2 - 1$

$du = \frac{1}{2\sqrt{x+1}}\,dx$

$2du = \frac{dx}{\sqrt{x+1}}$

$= 2\int (u^2-1)\,du$

$= 2\left(\frac{1}{3}u^3 - u\right)$

$\boxed{= \frac{2(\sqrt{x+1})^3}{3} - 2(\sqrt{x+1}) + C}$

Same answer
different subtitution

Again

$\int x^3 \cdot \sqrt{x^2+1}\, dx$

$u = \sqrt{x^2+1} \rightarrow x^2 = u^2 - 1$

$du = \dfrac{2x}{2\sqrt{x^2+1}}\, dx$

$2u\,du = 2x\,dx$

$= \int (u^2-1)\, u \cdot u\, du$

$= \int (u^4 - u^2)\, du$

$= \dfrac{1}{5} u^5 - \dfrac{1}{3} u^3 + C$

$$\boxed{= \frac{(\sqrt{x^2+1})^5}{5} - \frac{(\sqrt{x^2+1})^3}{3} + C}$$

Same answer as before

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Definite Integrals

$$\int_{x=a}^{x=b} f(g(x)) \cdot g'(x)\, dx$$

$$= F(g(x))\Big|_{x=a}^{x=b}$$

$$= F(g(b)) - F(g(a))$$

$$= F(u)\Big|_{u=g(a)}^{u=g(b)}$$

$$= \int_{u=g(a)}^{u=g(b)} f(u)\, du$$

Change the limits Also

ex $\int_0^{\sqrt{7}} x(x^2+1)^{\frac{1}{3}}\,dx$

$u = x^2+1 \qquad x=\sqrt{7} \rightarrow u=8$

$du = 2x\,dx \qquad x=0 \rightarrow u=1$

$= \int_{u=1}^{u=8} \frac{1}{2} u^{\frac{1}{3}}\,du$

$= \frac{3}{8} u^{\frac{4}{3}} \Big|_{u=1}^{u=8}$

$= \frac{3}{8}(16-1) = \frac{45}{8}$

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$$\int_2^4 \frac{dx}{x\ln x}$$

$v = \ln x$
$dv = \frac{1}{x}\,dx$

$x=4 \to v=\ln 4$
$x=2 \to v=\ln 2$

$$= \int_{v=\ln 2}^{v=\ln 4} \frac{dv}{v}$$

$$= \ln|v| \Big|_{\ln 2}^{\ln 4}$$

$$= \boxed{\ln(\ln(4)) - \ln(\ln(2))}$$

$\ln 4 = \ln 2^2 = 2\ln 2$

$$= \ln\left(\frac{\ln 4}{\ln 2}\right) = \boxed{\ln 2}$$

$$\int_0^{\frac{\pi}{6}} (1-\cos 3t)\sin 3t\, dt$$

$v = 1-\cos 3t$
$dv = 3\sin 3t\, dt$

$t = \frac{\pi}{6} \to v = 1$
$t=0 \to v=0$

$$= \int_{v=0}^{v=1} \frac{1}{3} v\, dv$$

$$= \frac{v^2}{6}\Big|_0^1 = \frac{1^2}{6} - \frac{0^2}{6} = \frac{1}{6}$$

$$\int_{-1}^{0} \frac{x}{1+x^4}\, dx$$

$\cancel{u = 1+x^4}$ $\quad u = x^2 \quad x=0 \rightarrow u=0$

$\cancel{du = 4x^3 dx}$ $\quad du = 2x\,dx \quad x=-1 \rightarrow u=1$

☆ Never use $u=x$!!!

$$\int_{u=1}^{u=0} \frac{\frac{1}{2} du}{1+u^2}$$

$$= -\int_{u=0}^{u=1} \frac{\frac{1}{2} du}{1+u^2}$$

$$= -\frac{1}{2} \arctan u \Big|_0^1$$

$$= -\frac{1}{2}\left(\frac{\pi}{4} - 0\right) = -\frac{\pi}{8}$$

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$$\int_0^{\pi} \cos x \, e^{\sin^2 x} \, dx$$

$u = \sin x$ $\quad x = \pi \rightarrow u = 0$

$du = \cos x \, dx$ $\quad x = 0 \rightarrow u = 0$

$$= \int_{u=0}^{u=0} e^{u^2} \, du = \boxed{0}$$