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Math 142
08/30/22

## 5.2 Definite Integrals

The definite integral of $f(x)$ on $[a,b]$ is the limit over all partitions and any $c_k \in [y_{k-1}, y_k]$

$$\lim_{\max \Delta x_k \to 0} \sum_{k=1}^{n} f(c_k)\,\Delta x_k$$

Notation:

upper limit (bound)

variable of integration (dummy variable)

integral sign (stretched S)

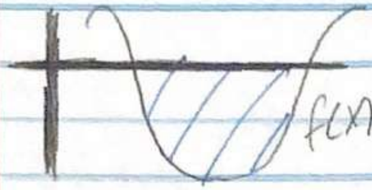
$$\int_a^b f(x)\,dx$$

Integrated

lower limit

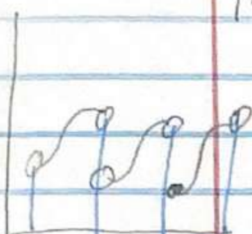
"the integral of $f(x)\,dx$"
"$f(x)$ is integrable"
- The definite integral is a number

ex $\int_0^1 x^2\,dx = \frac{1}{3}$




If f(x) is negative, area is negative.

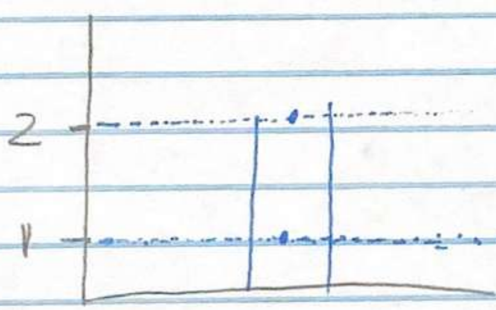
Thm. If f(x) is (piece wise) continuous, increasing, or decreasing on [a,b], It is intregrable on [a,b]



- If the limit exists, any sequence of partitions with max $\Delta X_K \to 0$ works

- right or left end points work

ex $f(x) = \begin{cases} 2, & x \text{ rational} \\ 1, & x \text{ irrational} \end{cases}$



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5.2 (con pt 2)

Integration Rules ( f, g integrable)

1. order of Integration $\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$
2. zero width interval $\int_a^a f(x)\,dx = 0$
3. constant multiple $\int_a^b k \cdot f(x)\,dx = k \cdot \int_a^b f(x)\,dx$
4. Sum/ difference $\int_a^b (f(x) \pm g(x))\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$
5. Adjacent intervals $\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$

Proof: Riemman Sums

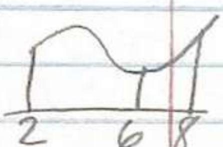
ex: let $\int_2^8 f(x)\,dx = 2$ $\int_6^8 f(x)\,dx = 3$

$\int_2^8 g(x)\,dx = 1$ $\int_6^8 g(x)\,dx = 5$

evaluate

$= \int_2^6 3f(x) - 2g(x)\,dx$

$= 3\int_2^6 f(x)\,dx - 2\int_2^6 g(x)\,dx$




$= 3(2-3) - 2(1-5)$

$= 9$

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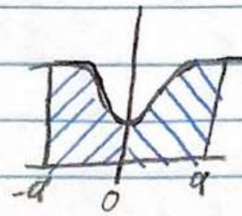
5.2 (con. pt3)

Symmertic interval $[-a,a]$

even function

-symmerty over y-Axis

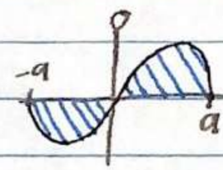



$f(-x) = f(x)$

$\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$

odd function

-symmerty about origin




$f(-x) = -f(x)$

$\int_{-a}^{a} f(x)\,dx = 0$

ex: $\int_{-\pi}^{\pi} (x^3+x)\,dx$ $\quad f(x) = x^3+x$ $\quad$ odd

$f(-x) = (-x)^3 - x = -f(x)$

$\boxed{=0}$

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ex: $\int_{-1}^{1} (x^2+\sin x)\,dx$

$f(x) = x^2+\sin x$

$f(-x) = x^2 - \sin x$ $\boxed{\text{Neither}}$

$= \int_{-1}^{1} x^2\,dx + \int_{-1}^{1} \sin x\,dx$

$= 2\int_{0}^{1} x^2\,dx + 0 = 2\cdot\frac{1}{3} = \boxed{\frac{2}{3}}$