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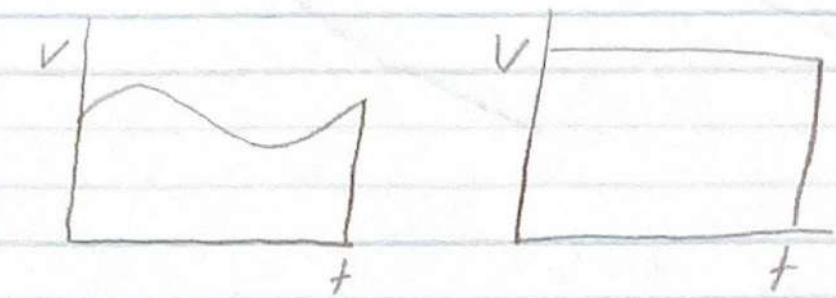
Math 142
8/26/22
Bickle

UHC 1357 2099

5.1 - Approximating Area pg0

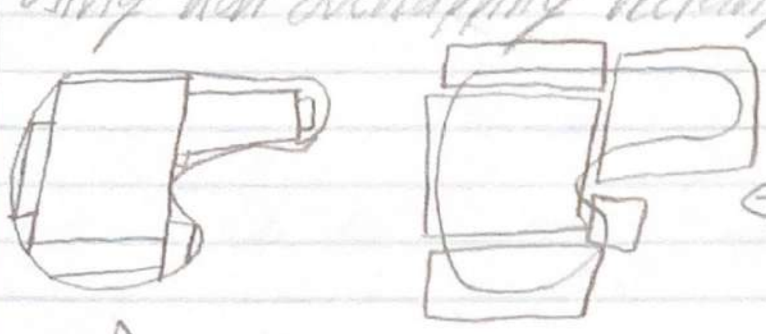
Given velocity, how to find distance traveled: (cont'd.)



$A = v \cdot t = \text{distance}$

Find area under the curve:
we know area formulas for specific shapes
not most curves

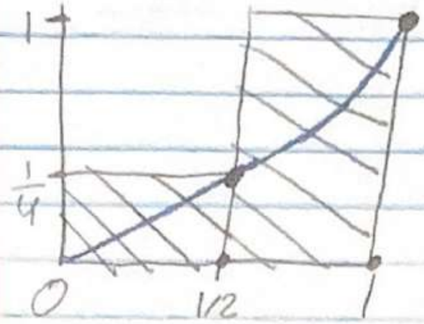
Define the area of a rectangle:
$(A = l \cdot w)$ and approximate more complicated shapes using non overlapping rectangles.

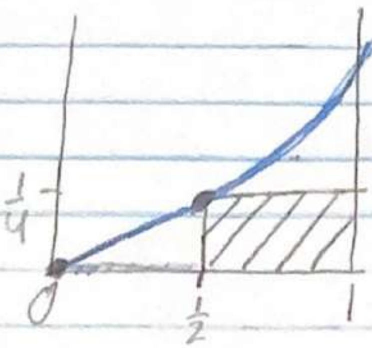


Rectangle completely inside shape $\Rightarrow$ area of shape bigger.

Rectangles completely cover shape $\Rightarrow$ area of shape smaller.

ex. $f(x) = x^2,\ 0 \leq x \leq 1$





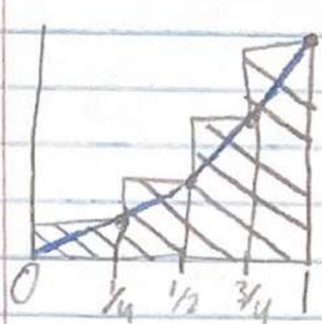
$n=2$

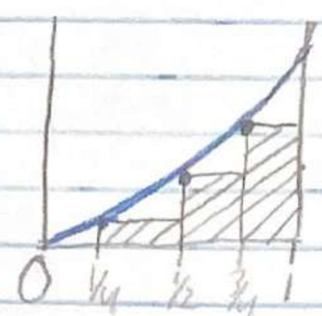
Right endpoints | left endpoints

$$\frac{1}{2}\cdot\frac{1}{4}+\frac{1}{2}\cdot 1 = \frac{1}{2}\left(\frac{1}{4}+1\right) = \frac{5}{8} > A$$

$$\frac{1}{2}\cdot 0 + \frac{1}{2}\cdot\frac{1}{4} = \frac{1}{8} < A$$

$$\frac{1}{8} < A < \frac{5}{8}$$





$n=4$

Right endpoints | Left endpoints

$$R_4 = \frac{1}{4}\left(\frac{1}{16}+\frac{1}{4}+\frac{9}{16}+1\right)$$

$$\frac{1}{4}\left(\frac{1}{16}+\frac{4}{16}+\frac{9}{16}+\frac{16}{16}\right) = \frac{15}{22} > A$$

$$L_4 = \frac{1}{4}\left(0+\frac{1}{16}+\frac{1}{4}+\frac{9}{16}\right)$$

$$\frac{1}{4}\left(\frac{0}{16}+\frac{1}{16}+\frac{4}{16}+\frac{9}{16}\right) = \frac{1}{4} < A$$

$$\frac{1}{4} < A < \frac{15}{22}$$

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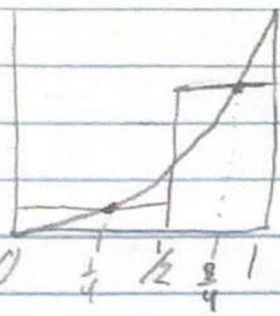
S.1 con. pg1

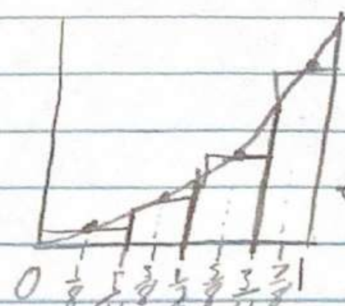
$n=10 \qquad 0.285 < A < 0.385$

$$R_{10} = \frac{1}{10}\left(\frac{1}{100} + \frac{4}{100} + \frac{9}{100} + \ldots \frac{81}{100} + \frac{100}{100}\right) = 0.385$$

$$L_{10} = \frac{1}{10}\left(0 + \frac{1}{100} + \frac{4}{100} + \ldots + \frac{81}{100}\right) = 0.285$$

Better estimates - Midpoint Rule





$$M_2 = \frac{1}{2}\cdot\frac{1}{16} + \frac{1}{2}\cdot\frac{9}{16} = \frac{1}{2}\left(\frac{1}{16} + \frac{9}{16}\right) = \frac{5}{16} = 0.3125$$

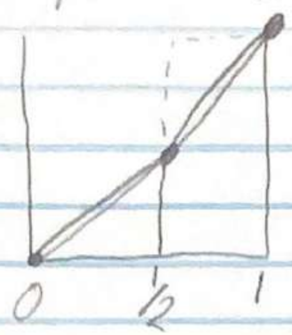
$$M_4 = \frac{1}{4}\left(\frac{1}{64} + \frac{9}{64} + \frac{25}{64} + \frac{49}{64}\right) = \frac{84}{266} = 0.328125$$

$$M_{10} = \frac{1}{10}\left(\frac{1}{400} + \ldots\right) = 0.3325$$

trapezoid rule

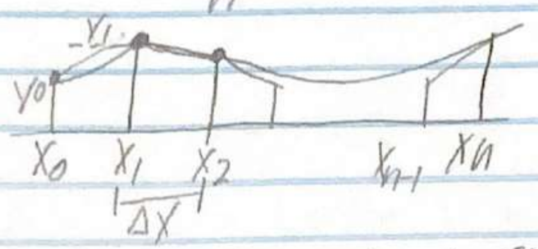
area est.

$$A_{trap} = \frac{h_1+h_2}{2} \cdot base$$



$$T_n = \frac{1}{2}\Delta x\,(y_0+2y_1+2y_2+\dots 2y_{n-1}+y_n)$$

$$\Delta x = \frac{b-a}{n} \text{ - base}$$



$$t_2 = \frac{0+\frac{1}{4}}{2}\cdot\frac{1}{2} + \frac{\frac{1}{4}+1}{2}\cdot\frac{1}{2} = \frac{1}{2}\cdot\frac{1}{2}\left(1\cdot 0+2\cdot\frac{1}{4}+1\cdot 1\right)$$

$$=\frac{3}{8} = 0.375$$

$$t_4 = \frac{1}{2}\cdot\frac{1}{4}\left(1\cdot 0+2\cdot\frac{1}{16}+2\cdot\frac{1}{4}+2\cdot\frac{9}{16}+1\cdot 1\right) = 0.34375$$

$$t_{10} = 0.335$$

what is the true area?

.3?

1/3?

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5.1 (con) pg 2

Summation
- need notation better than "..."

$$\left(\frac{1}{100}+\frac{4}{100}+\ldots+1\right)$$

Summation notation

Sigma for sum $\rightarrow$ $\sum_{i=1}^{n} a_i$ $\leftarrow$ formula for i^{th} term

$n \leftarrow$ index ends at n

$i=1 \leftarrow$ index starts at 1

index - integer-valued variable

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \ldots + a_n$$

$$\sum_{i=1}^{4} i^3 = 1^3+2^3+3^3+4^3 = 1+8+27+64 = 100$$

Example: express in Σ notation:

$2+5+8+11+14$ $\qquad \sum_{i=1}^{5}(3i-1) = \sum_{i=0}^{4}(3i+2)$

sum/diff. $\sum_{i=1}^{n}(a_i \pm b_i) = \sum_{i=1}^{n} a_i \pm \sum_{i=1}^{n} b_i$

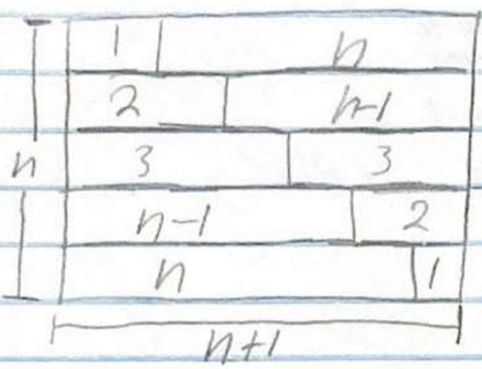
Constant mult. $\sum_{i=1}^{n} k \cdot a_i = k \sum_{i=1}^{n} a_i$

Constant value $\sum_{i=1}^{n} k = k \cdot n$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$$



Riemann Sums

- functions $f(x)$, interval $[a,b]$
- partition the interval
  - choose $a = x_0 < x_1 < x_2 < \dots < x_{n-1} < x_n = b$
  - break intervals into subintervals $[x_0, x_1], \dots, [x_{n-1}, x_n]$
- choose arbitary points c_i in each subinterval
- find area of rectangle $f(c_i) \cdot \Delta x_i$
- Riemann Sum
  - left RS
  - right RS $\qquad \sum_{i=1}^{n} f(c_i) \cdot \Delta x$
  - midpoint RS

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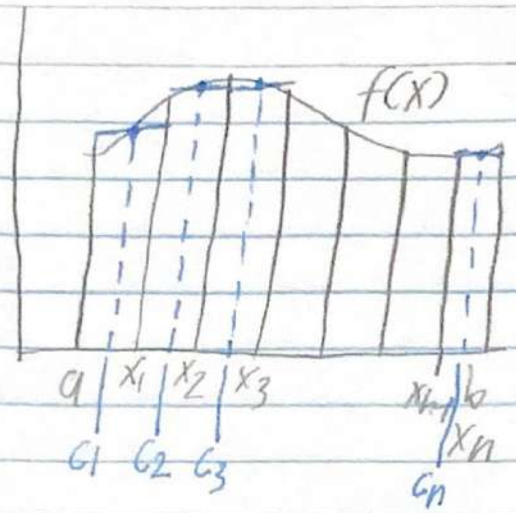
5.1 (con pg3)



"Slice and Sum"

equal length subintervals

$\sum_{i=1}^{n} f(c_i \cdot \Delta x)$

Riemmann Sum

$\Delta x = \frac{b-a}{n}$

- Left RSn $c_i = a + (i-1)\Delta x$
- Right RS $c_i = a + i \cdot \Delta x$
- Midpoint RS $c_i = a + (i - \frac{1}{2})\Delta x$

take limit as $\max(\Delta x_i) \to 0$ (as $n \to \infty$)

Def. If $\lim_{\Delta x \to 0} \sum_{i=1}^{n} f(c_i)\Delta x_i$ exists, it is the area under f(x) on [a, b]

definite integral

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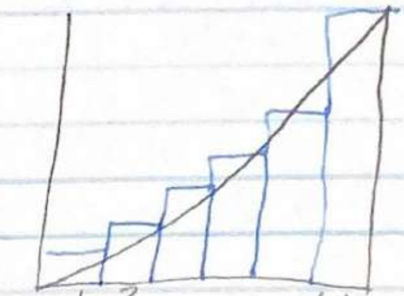
Area under $f(x)$

$$\lim_{\Delta x \to 0} \sum_{k=1}^{n} f(c_k)\,\Delta x_k$$



ex $f(x)=x^2$, $[0,1]$

$$\Delta x = \frac{1-0}{n}$$

$$R_n = \sum_{k=1}^{n} f\left(\frac{k}{n}\right)\cdot \frac{1}{n}$$

$$= \sum_{k=1}^{n} \left(\frac{k}{n}\right)^2 \cdot \frac{1}{n} \qquad c_k = a + k\cdot\Delta x = 0 + k\,\frac{1}{n}$$

$$= \frac{1}{n^3}\sum_{k=1}^{n} k^2$$

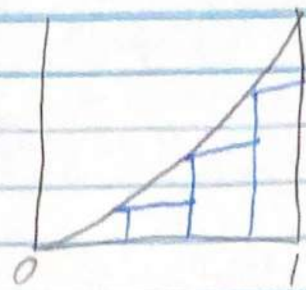
$$= \frac{1}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}$$

$\frac{\infty}{\infty}$

$$A_R = \lim_{n\to\infty} \frac{n(n+1)(2n+1)}{6n^3}$$

$$= \lim_{n\to\infty} \frac{2n^3 \ldots}{6n^3} = \frac{2}{6} = \frac{1}{3}$$

$$L_n = \sum_{k=0}^{n-1} f\left(\frac{k}{n}\right)\frac{1}{n}$$



$$= \sum_{k=0}^{n-1} \frac{k^2}{n^2}\cdot\frac{1}{n}$$

$$= \frac{1}{n^3}\sum_{k=1}^{n-1} k^2$$

$$= \frac{1}{n^3}\cdot\frac{(n-1)n(2n-1)}{6}$$

$$A_L = \lim_{n\to\infty}\frac{(n-1)n(2n-1)}{6n^3} = \frac{1}{3}$$

The area is $\frac{1}{3}$! (by definition)