

Homework 7 (ECE 313/ECE 317, Fall 2023):

Problem 1:

Consider the following table of joint PMF of the two random variables X and Y

$X, /Y$	1	2	3	$\mathbb{P}_X(x_i)$
1	0.2	0.1	0.25	
2	0.1	0.04	0.11	
3	0	0.1	0.1	
$\mathbb{P}_Y(y_i)$				

- 1) Find the marginal probabilities of each random variable X and Y.
- 2) Compute the expectation and the variance of each random variable
- 3) Compute the covariance and the correlation of X and Y.
- 4) Are X and Y independent?

Problem 2:

Suppose you take data of stock returns from the Excelsior Corporation and the Adirondack Corporation from the years 2008 to 2012, as shown in the following table (X represents the returns to Excelsior and Y represents the returns to Adirondack):

Year	Excelsior Corp. Annual return X	Excelsior Corp. Annual return Y
2008	1	3
2009	-2	2
2010	3	4
2011	0	6
2012	3	0

- 1) What are the expectation and variance of each random variable.
- 2) What are the covariance and correlation between the stock returns (use the formula of the population covariance and correlation)?

Problem 3: Consider the joint PMF of the random variables X and Y

X/Y	-1	0	1	$\mathbb{P}_X(x_i)$
-1	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{4}$	
0	$\frac{1}{18}$	$\frac{1}{9}$	$\frac{1}{6}$	
1	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{12}$	
$\mathbb{P}_Y(y_i)$				

- 1) Compute the marginal probabilities

- 2) Compute $\mathbb{E}[X]$, $\mathbb{E}[Y]$ and $\mathbb{E}[XY]$.
- 3) Are X and Y independent?
- 4) Compute the correlation of X and Y
- 5) Compute the conditional probabilities : $\mathbb{P}(X = x_i|Y = y_i)$ for all x_i and y_i
- 6) Compute $\mathbb{P}(x \leq 0|Y \geq 0)$
- 7) Compute $\mathbb{P}(X + Y = 0)$
- 8) Compute $\mathbb{E}[-X + 4Y]$ and $V(X + Y)$

Problem 4:

A) Let X_i be a Bernoulli distribution with parameter $p = \frac{1}{3}$ and let $S_n = X_1 + \dots + X_{25} \rightarrow \mathcal{B}(25, \frac{1}{3})$ (follows a binomial distribution with $n = 25$ and $p = \frac{1}{3}$).

- 1) Compute $\mathbb{E}[S_n]$ and $V(S_n)$
- 2) Compute the following probabilities: $\mathbb{P}(S_n \leq 5)$, $\mathbb{P}(S_n \leq 8)$ and $\mathbb{P}(S_n \leq 15)$

B) Suppose the grades in a finite mathematics class are Normally distributed with a mean of 70 and a standard deviation of 6 $\rightarrow \mathcal{N}(70, 6^2)$

- a) What is the probability that a randomly selected student had a grade of at least 75?
- b) What is the probability that the average grade for 10 randomly selected students was at least 75?
- c) What is the probability that the average grade for 25 randomly selected students was at least 75?