Homework3 (ECE 313/ ECE 317), Fall 2023

Problem 1:
In a population Ω, two diseases D_1 and D_2 are present in 40% and 20% respectively. It is assumed that the number of those who suffer from both diseases is negligible (i.e; $D_1 \cap D_2 = \emptyset$). We undertake a screening system of D_1 and D_2 diseases. To do this, we apply a test that works on 90% of D_1 patients, on 70% of D_2 patients, and in 10% of individuals who have neither of these two conditions.

1) When we randomly choose an individual ω in Ω, what is the probability that the test will work?

2) Knowing that for an individual ω, the test reacted, give the probabilities:
   • So that the test reacted because of the D_1 disease.
   • So that the test reacted because of the D_2 disease.
   • So that the test reacted when the individual was infected with neither of the two diseases D_1 and D_2.

Problem 2: (Classification using probabilities)
One way to design a spam filter is to look at the words in an email. In particular, some words are more frequent in spam emails. Suppose that we have the following information:
• 40% of emails are spam.
• 9% of spam emails contain the word "refinance".
• 0.01% of non-spam emails contain the word "refinance".
- Suppose that an email is checked and found to contain the word "refinance". What is the probability that the email is spam?

Problem 3:
A tennis player is entitled to two attempts to make a successful throw-in.
The player succeed his first serve 65% of the time. When it fails, it succeeds the second in 90% of cases.
- What is the probability that the player will double fault (i.e. fails twice in a row)?

Problem 4:
Suppose that I have three bags that each contain 100 marbles:

- Bag B_1 has 80 red and 20 blue marbles;

- Bag B_2 has 65 red and 35 blue marbles;

- Bag B_3 has 45 red and 55 blue marbles.

I choose one of the bags at random and then pick a marble from the chosen bag, also at random.

1) What is the probability that the chosen marble is red?

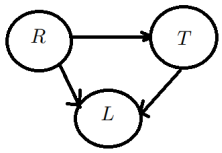

Figure 1

2) Suppose we observe that the chosen marble is red. What is the probability that Bag B_1 was chosen?

3) Is the picked marble being red depend on the chosen bag? (check the independence of the two events using the conditional probability)

Problem 5:
In my town, it's rainy one third of the days. Given that it is rainy, there will be heavy traffic with probability $\frac{1}{2}$, and given that it is not rainy, there will be heavy traffic with probability $\frac{1}{4}$. If it's rainy and there is heavy traffic, I arrive late for work with probability $\frac{1}{2}$. On the other hand, the probability of being late is reduced to $\frac{1}{8}$ if it is not rainy and there is no heavy traffic. In other situations (rainy and no traffic, not rainy and traffic) the probability of being late is 0.25. The network connection is present in Figure 1. You pick a random day.

1) What is the probability that it is raining and there is heavy traffic and I am late?

2) What is the probability that it's not raining and there is heavy traffic and I am not late?

3) What is the probability that I am late?

4) Given that I arrived late at work, what is the probability that it rained that day?