

## Homework 2 (ECE 313/ ECE317, Fall 2023):

### Problem 1:

Express each of the following events in terms of events  $A$ ,  $B$ , and  $C$  as well as of set's operations (complement, union, and intersection):

- 1) At least one of the events  $A$ ,  $B$ ,  $C$  occurs.
- 2) At most one of the events  $A$ ,  $B$ ,  $C$  occurs.
- 3) None of the events  $A$ ,  $B$ ,  $C$  occurs.
- 4) All the three events  $A$ ,  $B$ ,  $C$  occur.
- 5) Exactly one of the events  $A$ ,  $B$ ,  $C$  occurs.
- 6) Event  $A$  and  $B$  occur, but not  $C$ .
- 7) Event  $A$  and  $B$  doesn't occur and  $C$  occurs.
- 8) event  $A$  occurs or, if not, then  $B$  also doesn't occur.

### Problem2:

A) Let  $A$  and  $B$  be two events such that:  $\mathbb{P}(B) = 0.4$ ,  $\mathbb{P}(A \cup B) = 0.9$ . Answer to the following questions:

- 1) Compute  $\mathbb{P}(A - B)$ .
- 2) If  $\mathbb{P}(A \cap B) = 0.1$ , what is  $\mathbb{P}(A)$  ?
- 3) Compute  $\mathbb{P}[(A \cap B)^c]$ .
- 4) Compute  $\mathbb{P}(A^c \cap B^c)$ .

B) Let  $A$  and  $B$  be two events. Use the axioms of probability to prove the following:

- 1)  $\mathbb{P}(A \cap B) \geq \mathbb{P}(A) + \mathbb{P}(B) - 1$ .
- 2)  $\mathbb{P}(A \cap B \cap C) \geq \mathbb{P}(A) + \mathbb{P}(B) + \mathbb{P}(C) - 2$ .
- 3) The probability that one and only one of the events  $A$  or  $B$  occurs is  $\mathbb{P}(A) + \mathbb{P}(B) - 2\mathbb{P}(A \cap B)$ .

### Problem3:

You flip a fair coin three times.

- 1) What is the sample set of all the possible outcomes?
- 2) What is the probability of each outcome?
- 3) What is the probability that exactly one of the flips results in a Tail?
- 4) What is the probability to get at least two Heads?

- 5) What is the probability to get at most one Tail?
- 6) What is the probability to get the first flipping Head?

**Problem4:**

A rectangular LCD screen has dimensions of  $60\text{cm} \times 45\text{cm}$ . Noting all pixels by  $x$  and  $y$  coordinates such that  $0 \leq x \leq 45$  and  $0 \leq y \leq 60$ . Knowing that a pixel is defective, determine the probability of the following events:

- 1) The defective pixel is in the area determined by  $20 \leq x \leq 30$  and  $10 \leq y \leq 50$
  - 2) The defective pixel is in the area determined by  $x \geq 38$
  - 3) The defective pixel is in the area determined by  $x \geq 38$  and  $y \geq 52$
  - 4) The defective pixel is in the area determined by  $(x \leq 20$  or  $x \geq 30)$  and  $(y \leq 10$  or  $y \geq 50)$
  - 5) The defective pixel is in the area determined by  $y - x \geq 10$ .
- Plot the above areas.  
- Indication: It is a continuous probability law.

**Problem5:**

In a group of 40 people, 10 people are interested in fishing, 15 in reading and 18 are not interested in fishing or reading. We randomly select a person from the group. Calculate the probability for that person to be:

- 1) Interested in fishing.
- 2) Interested in reading.
- 3) Not interested to neither of the two activities
- 4) Interested in at least one of the two activities.
- 5) Interested in both activities.
- 6) Interested in only reading.
- 7) Interested in only fishing.