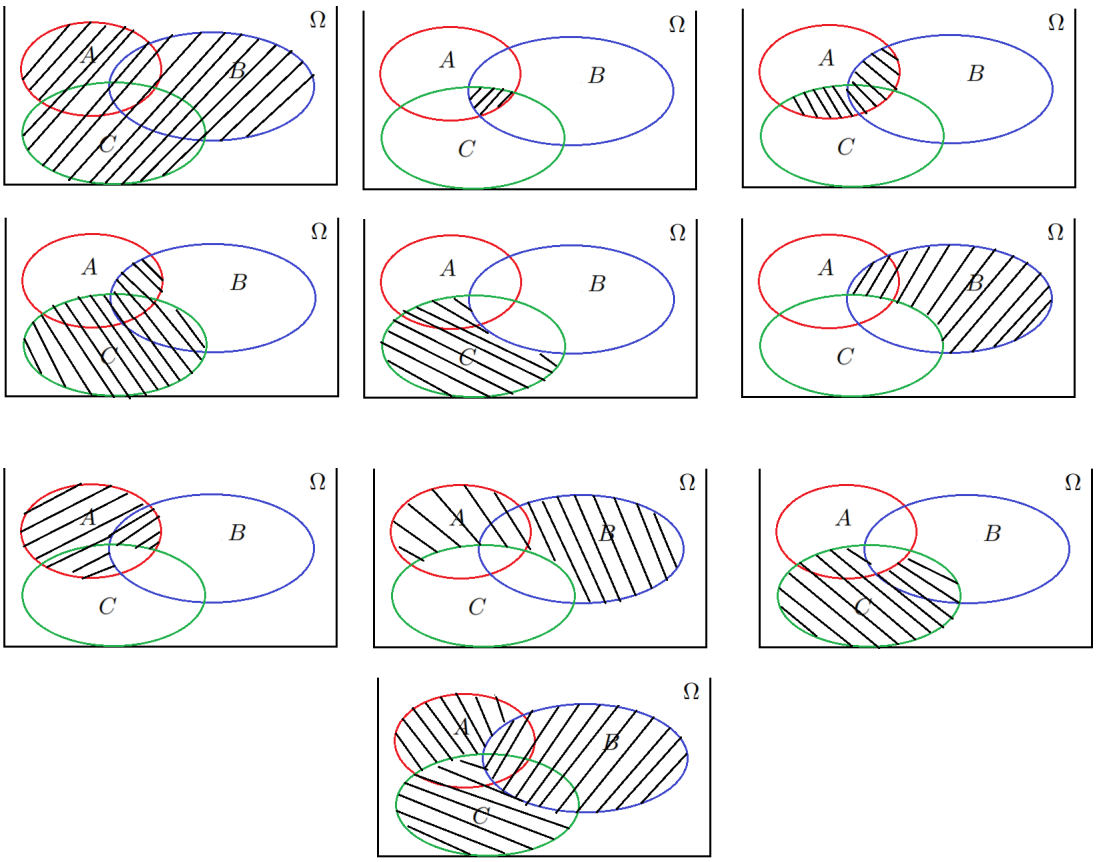

HW1-Solution (ECE 313/ECE 317, Fall 2023):

Problem 1: (5 pts)
Let A, B, C be three sets as shown in Figure 1 (Venn diagram). For each of the following sets, shade the area representing the given sets.

1) $A \cup B \cup C$ 0.5

2) $A \cap B \cap C$ 0.5

3) $A \cap (B \cup C)$ 0.5

4) $C \cup (A \cap B)$ 0.5

5) $C - B$ 0.5

6) $B - C$ 0.5

7) $A - (B \cap C)$ 0.5

8) $(A \cup B) - C$ 0.5

9) $C \cap (A \cap B)^c$ 0.5

10) $B \cup (C - B) \cup [A - (C \cup B)]$ 0.5

Problem 2: (5 pts)
1) Let the subsets $A = \{11, 80, 10, 19, 14\}$, $B = \{10, 14, 20\}$ and $C = \{20, 10, 11, 5\}$ of the set $S = \{11, 80, 5, 10, 19, 3, 14, 20\}$.
-Compute the following sets:

1) $A \cup B \cup C = \{11, 80, 10, 19, 14, 20, 5\}$ 0.5

2) $A \cap B \cap C = \{10\}$ 0.5

3) $(A \cap B \cap C)^c = \{11, 80, 5, 19, 3, 14, 20\}$ 0.5

4) $(A \cup B \cup C)^c = \{3\}$ 0.5

5) $A - (C \cap B) = \{11, 80, 19, 14\}$ 0.5

6) $C - (A \cup B) = \{5\}$ 0.5

2) Using the DeMorgan's laws verify the following equalities: We have
$A^c = \{20, 5, 3\}$ 0.25
$B^c = \{11, 80, 5, 19, 3\}$ 0.25
$C^c = \{80, 19, 3, 14\}$ 0.25

- $(A \cup B \cup C)^c = A^c \cap B^c \cap C^c$

  $$(A \cup B \cup C)^c = \{3\} \quad \text{and} \quad A^c \cap B^c \cap C^c = \{3\} \qquad 0.25$$

- $[(A \cap B) \cup C]^c = (A^c \cup B^c) \cap C^c$
  - We have
  $A \cap B = \{10, 14\}$ 0.25
  $(A \cap B) \cup C = \{20, 10, 11, 5, 14\}$ 0.25
  $\Rightarrow [(A \cap B) \cup C]^c = \{80, 19, 3\}$ 0.25
  And
  $A^c \cup B^c = \{5, 3, 20, 11, 80, 19\} \Rightarrow (A^c \cup B^c) \cap C^c = \{80, 19, 3\}$ 0.25

Problem 3: (4 pts)
In a sample of 200 persons, 140 like tea, 95 like coffee, and 11 like neither. Let
A= the set of people who like tea.
B=the set of people who like coffee.
C=the set of people who like neither.

$$n(S) = 200, \quad n(A) = 140, \quad n(B) = 95, \quad n(C) = 11$$

1) How many people like tea or coffee? $n(A \cup B) =?$

$$n(A \cup B) = 200 - 11 = 189 \qquad 0.5$$

2) How many people like both? $n(A \cap B) = ?$

$$n(A \cup B) = n(A) + n(B) - n(A \cap B) \Rightarrow 189 = 140 + 95 - n(A \cap B)$$

$$\Rightarrow n(A \cap B) = 140 + 95 - 189 = 235 - 189 = 46 \qquad 0.5$$

3) How many people like coffee only? $n(B - A) = ?$

$$n(B) = n(B-A)+n(A \cap B) \Rightarrow 95 = n(B-A)+46 \Rightarrow n(B-A) = 95-46 = 49 \qquad 0.5$$

4) How many people like tea only? $n(A - B) = ?$

$$n(A) = n(A-B)+n(A \cap B) \Rightarrow 140 = n(A-B)+46 \Rightarrow n(A-B) = 140-46 = 94 \qquad 0.5$$

5) How many people don't like tea? $n(A^c) = ?$

$$n(A^c) = n(S) - n(A) \Rightarrow n(A^c) = 200 - 140 = 60 \qquad 0.5$$

6) How many people don't like coffee? $n(B^c) = ?$

$$n(B^c) = n(S) - n(B) \Rightarrow n(B^c) = 200 - 95 = 105 \qquad 0.5$$

7) How many people don't like both coffee and tea? $n[(A \cap B)^c] = ?$

$$n[(A \cap B)^c] = n(S) - n(A \cap B) = 200 - 46 = 154 \qquad 0.5$$

8) How many people don't like coffee and don't like tea? $n[A^c \cap B^c] = ?$

$$n[A^c \cap B^c] = n[(A \cup B)^c] = n(S) - n(A \cup B) = 200 - 189 = 11 \qquad 0.5$$

Problem 4: (6 pts)

A) Write in different ways the following sets:

1) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ 0.5

2) $A - B = A \cap B^c$ 0.5

3) $(A - B) \cap (A - C) = (A \cap B^c) \cap (A \cap C^c) = A \cap B^c \cap C^c$ 0.5

4) $A - (B \cup C) = A \cap (B \cup C)^c = A \cap B^c \cap C^c = A \cap B^c \cap C^c$ 0.5

5) $[A \cap (B^c \cap C^c)]^c = A^c \cup (B^{cc} \cup C^{cc}) = A^c \cup B \cup C$ 0.5

6) $[(A \cap B) \cup (B \cap C)^c]^c = (A \cap B)^c \cap (B \cap C)^{cc} = (A^c \cup B^c) \cap (B \cap C)$ 0.75

$$= [A^c \cap (B \cap C)] \cup (\underbrace{\underbrace{B^c \cap B}_{\emptyset} \cap C}_{\emptyset}) = A^c \cap B \cap C$$

7) $(A \cap B) \cup (A - B) = (A \cap B) \cup (A \cap B^c) = A \cap (B \cup B^c) = A$ 0.5

8) $A \cup B \cup C = A \cup (B - A) \cup [C - (A \cup B)] = A \cup (B \cap A^c) \cup (C \cap A^c \cap B^c)$
or

$A \cup B \cup C = B \cup (A - B) \cup [C - (A \cup B)] = B \cup (A \cap B^c) \cup (C \cap A^c \cap B^c)$

or

$A \cup B \cup C = C \cup (B - C) \cup [A - (C \cup B)] = C \cup (B \cap C^c) \cup (A \cap C^c \cap B^c)$ 0.75

9) $A \cup (B^c \cup C^c)^c = A \cup (B^{cc} \cap C^{cc}) = A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
0.5

B) Using the answer of 9), find $A \cup (B^c \cup C^c)^c$ in each of the following cases:

(a) $B \cap C = \emptyset \Rightarrow A \cup (B^c \cup C^c)^c = A \cup \underbrace{(B \cap C)}_{\emptyset} = A$ 0.5

(b) $A \subset B \Rightarrow A \cup (B^c \cup C^c)^c = \underbrace{(A \cup B)}_{=B} \cap (A \cup C) = B \cap (A \cup C)$ 0.5