

## Midterm Exam ECE 313/ECE317, Fall 2023

### Problem 1:

Suppose we are about to roll a balanced six-sided die once. Let the sample set  $\Omega = \{1, 2, 3, 4, 5, 6\}$ , and the following events

$A = \{2, 4, 6\} \rightarrow$  Getting an even number

$B = \{1, 3, 5\} \rightarrow$  Getting an odd number

$C = \{3, 6\}$

1) Determine the outcomes of each of the following events:

- a) Events A and C occur
- b) Events B and C occur
- c) Events A and B and C occur
- e) Event A occurs and not B and not C

2) Compute the following probabilities:

$$\mathbb{P}(A), \mathbb{P}(B), \mathbb{P}(C), \mathbb{P}(A \cap C), \mathbb{P}(B \cap C), \mathbb{P}(A \cap B \cap C)$$

3) Compute the probabilities:  $\mathbb{P}(A|C)$ ,  $\mathbb{P}(B|C)$ ,  $\mathbb{P}((A \cap B)|C)$  and answer to the following questions:

- a) Are A and C independent (i.e;  $\mathbb{P}(A|C) = \mathbb{P}(A)$ ) ?
- b) Are B and C independent (i.e;  $\mathbb{P}(B|C) = \mathbb{P}(B)$ )?
- c) Are A and B independent (i.e;  $\mathbb{P}((A \cap B)) = \mathbb{P}(A) \cdot \mathbb{P}(B)$ )?
- d) Are A and B conditionally independent knowing C (i.e;  $\mathbb{P}((A \cap B)|C) = \mathbb{P}(A|C) \cdot \mathbb{P}(B|C)$ )?

### Problem 2:

Sara and Bob each choose at random a number in the **interval**  $[1, 5]$ . Consider the following events: (Indication: It is a continuous probability law)

- A) Both numbers are greater than 2.8
- B) Both numbers are less than 3.7
- C) Sara's number is greater than 3.7

- Plot the above areas.

- Find the probabilities:  $\mathbb{P}(A)$ ,  $\mathbb{P}(B)$ ,  $\mathbb{P}(C)$ ,  $\mathbb{P}(A|C)$

### Problem 3:

In a group of sick persons, two diseases  $D_1$  and  $D_2$  are present in 55% and 45% respectively. We gave a drug M to this group of sick persons, and we remarked that 53% from patients having  $D_1$  are relieved and 61% from patients having  $D_2$  are relieved

1. When we randomly choose an individual from this group, what is the probability that this person relieved?
2. What is the likelihood of a patient to have  $D_1$  knowing that he is relieved?
3. What is the likelihood of a patient to have  $D_2$  knowing that he is relieved?

**Problem 4:**

A study investigated causes of sudden death in one area. A sample of 523 such death revealed the following

	Cardiovascular(CV)	Cerebral(C)	Respiratory(R)	Other	Total
Males (M)	264	38	36	21	359
Females (F)	89	27	29	19	164
Total	353	65	65	40	523

- 1) Suppose one of these cases is randomly selected. What is the probability the person was female?
- 2) What is the probability that the cause was cardiovascular ?
- 3) Given the cause was cardiovascular in nature, what is the probability the person was female (i.e  $\mathbb{P}(F|CV)$ )?