Midterm Exam-Solution ECE 313/ECE317, Fall 2023

Problem 1: (5.75 points)
Suppose we are about to roll a balanced six-sided die once. Let the sample set $\Omega = \{1, 2, 3, 4, 5, 6\}$, and the following events
$A = \{2, 4, 6\}$
$B = \{1, 3, 5\}$
$C = \{3, 6\}$

1) Determine the outcomes of each of the following events:

a) A and C occur $\rightarrow$ $A \cap C = \{6\}$ (0.25)

b) B and C occur $\rightarrow$ $B \cap C = \{3\}$ (0.25)

c) A and B and C occur $\rightarrow$ $A \cap B \cap C = \emptyset$ (0.25)

d) A occurs and not B and not C $\rightarrow$ $A \cap B^c \cap C^c = A - (B \cup C) = \{2, 4\}$ (0.25)

2) Compute the following probabilities:

$$\mathbb{P}(A) = \frac{1}{2} \quad (0.25) \qquad \mathbb{P}(A \cap C) = \frac{1}{6} \quad (0.5)$$
$$\mathbb{P}(B) = \frac{1}{2} \quad (0.25) \qquad \mathbb{P}(B \cap C) = \frac{1}{6} \quad (0.5)$$
$$\mathbb{P}(C) = \frac{1}{3} \quad (0.25) \qquad \mathbb{P}(A \cap B \cap C) = 0 \quad (0.5)$$

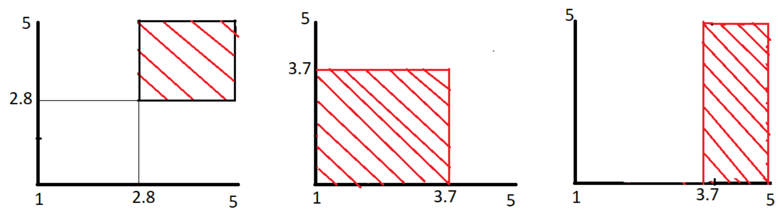
3) Compute the probabilities: $\mathbb{P}(A|C)$, $\mathbb{P}(B|C)$, $\mathbb{P}((A \cap B)|C)$ and answer to the following questions:

$$\mathbb{P}(A|C) = \frac{\mathbb{P}(A \cap C)}{\mathbb{P}(C)} = \frac{1}{2} \quad (0.5)$$
$$\mathbb{P}(B|C) = \frac{\mathbb{P}(B \cap C)}{\mathbb{P}(C)} = \frac{1}{2} \quad (0.5)$$
$$\mathbb{P}((A \cap B)|C) = \frac{\mathbb{P}(A \cap B \cap C)}{\mathbb{P}(C)} = 0 \quad (0.5)$$

a) Are A and C independent:$\rightarrow$ Yes, since $\mathbb{P}(A|C) = \frac{1}{2} = \mathbb{P}(A)$ (0.25)

b) Are B and C independents: ($\rightarrow$ Yes, since $\mathbb{P}(B|C) = \frac{1}{2} = \mathbb{P}(B)$ (0.25)

c) Are A and B independent: $\rightarrow$ No, since $\mathbb{P}(A \cap B) = 0 \neq \mathbb{P}(A).\mathbb{B} = \frac{1}{4}$ (0.25)



(0.75) → (0.25) for each plot

d) Are A and B conditionally independent knowing C: → No, since
$\mathbb{P}((A \cap B)|C) = 0 \neq \mathbb{P}(A|C).\mathbb{P}(B|C) = \frac{1}{4}$ (0.25)

Problem 2: (4.75 points)
Sara and Bob each choose at random a number in the interval $[1, 5]$. Consider the following events:(Indication: It is a continuous probability law and not discrete $---->$ you should consider the area)

$$\text{Area}(\Omega) = 4 \times 4 = 16$$

A) Both numbers are greater than 2.8 → $Area(A) = (5-2.8)\times(5-2.8) = 4.84$

$$\mathbb{P}(A) = \frac{area(A)}{area(\Omega)} = \frac{4.84}{16} = 0.3025 \qquad (01)$$

B) Both numbers are less than 3.7 → $Area(B) = (3.7-1)\times(3.7-1) = 7.29$

$$\mathbb{P}(B) = \frac{area(B)}{area(\Omega)} = \frac{7.29}{16} = 0.4556 \qquad (01)$$

C) Sara's number is greater than 3.7 → $Area(C) = (5-3.7)\times 4 = 5.2$

$$\mathbb{P}(C) = \frac{area(C)}{area(\Omega)} = \frac{5.2}{16} = 0.32 \qquad (01)$$

$$\mathbb{P}(A|C) = \frac{\mathbb{P}(A \cap C)}{\mathbb{P}(C)} = \frac{area(A \cap C)}{area(C)} = \frac{(5-2.8)\times(5-3.7)}{5.2} = 0.55$$

(01)

- Plot the above areas.

Problem 3: (5 points)
In a group of sick persons, two diseases D_1 and D_2 are present at 55% and 45% respectively. We gave a drug M to this group of sick persons, and we remarked that 53% from patients having D_1 are relieved and 61% of patients having D_2 are relieved. We have the following probabilities

$$\mathbb{P}(D_1) = 0.55, \quad \mathbb{P}(D_2) = 0.45, \quad \mathbb{P}(R|D_1) = 0.53, \quad \mathbb{P}(R|D_2) = 0.61$$

1. When we randomly choose an individual from this group, what is the probability that this person relieved?

$$\begin{aligned}\mathbb{P}(R) &= \mathbb{P}(R \cap D_1) + \mathbb{P}(R \cap D_2) \\ &= \mathbb{P}(R|D_1).\mathbb{P}(D_1) + \mathbb{P}(R|D_2).\mathbb{P}(D_2) \\ &= (0.53).(0.55) + (0.61).(0.45) \\ &= 0.566 \qquad (1.5)\end{aligned}$$

2. What is the likelihood that a patient has D_1 knowing that this person has been relieved?

$$\mathbb{P}(D_1|R) = \frac{\mathbb{P}(D_1 \cap R)}{\mathbb{P}(R)} = \frac{\mathbb{P}(R|D_1).\mathbb{P}(D_1)}{\mathbb{P}(R)} = \frac{(0.53).(0.55)}{0.566} = 0.515 \qquad (1.5)$$

3. What is the likelihood that a patient has D_2 knowing that this person has been relieved?

$$\mathbb{P}(D_2|R) = \frac{\mathbb{P}(D_2 \cap R)}{\mathbb{P}(R)} = \frac{\mathbb{P}(R|D_2).\mathbb{P}(D_2)}{\mathbb{P}(R)} = \frac{(0.61).(0.45)}{0.566} = 0.485 \qquad (1.5)$$

4) This drug works better for which disease?
- The drug works better for disease D_1 (0.5)

Problem 4: (4.5 points)
A study investigated causes of sudden death in one area. A sample of 523 such death revealed the following

	Cardiovascular(CV)	Cerebral(C)	Respiratory(R)	Other	Total
Males (M)	264	38	36	21	359
Females (F)	89	27	29	19	164
Total	353	65	65	40	523

1) Suppose one of these cases is randomly selected. What is the probability that the person was female?

$$\mathbb{P}(F) = \frac{|F|}{|\Omega|} = \frac{164}{523} = 0.3136 \qquad 1.5$$

2) What is the probability that the cause was cardiovascular ?

$$\mathbb{P}(CV) = \frac{|CV|}{|\Omega|} = \frac{353}{523} = 0.675 \qquad 1.5$$

3) Given the cause was cardiovascular in nature, what is the probability the person was female (i.e $\mathbb{P}(F|CV)$)?

$$\mathbb{P}(F|CV) = \frac{\mathbb{P}(F \cap CV)}{\mathbb{P}(CV)} = \frac{|F \cap CV|}{|CV|} = \frac{89}{353} = 0.2521 \qquad 1.5$$