Final Exam (ECE 313, Fall 2023):

Problem1:
It is known that 25% of products on a production line are defective. Products are inspected **until** first defective is encountered. Let X = number of inspections to obtain first defective ($X \to$ Geometric distribution with parameter $p = 0.25$)

1) What is the probability that the first defective occurs in the sixth item inspected?
2) What is the average number of inspections to obtain the first defective?
3) What is the probability that the first defective occurs in the first five inspections? (i.e $\mathbb{P}(X \leq 5)$)

Problem2:
A technology company uses the Poisson distribution to model the number of expected network failures per month. It has been detected that, on average, there are 3 network failures per two months (we consider here that there are 60 days in 2 months)

1) What is the probability that the company experiences 2 of network failures in a given two months (60 days).
2) What is the probability that the company experiences between 2 to 5 network failures in a given two months (60 days).
3) On average, how many days elapse between two failures? .
4) Note that the waiting time that a network fails follows an exponential distribution with a decay parameter $\lambda = \frac{1}{20}$. What is the probability that a network fails within 16 days?

Problem3:
The distribution of retirement age for NFL players is normally distributed with a mean of 35 years old and a standard deviation of about 1.5 years.

1) Find the probability that a randomly selected player will be retired at an age less than 33 years old?
2) Find the probability that a randomly selected player will be retired between 33 and 35 years old.

Problem4:
A) Suppose you take a stock returns from the Excelsior Corporation and the Adirondack Corporation from the years 2008 to 2012, as shown here (X represents the returns to Excelsior and Y represents the returns to Adirondack):

Year	Excelsior Corp. Annual return X	Excelsior Corp. Annual return Y
2008	1	3
2009	-2	2
2010	3	4
2011	1	5
2012	2	1

1) What are the expectation and variance of each random variable.

2) What are the covariance and correlation between the stock returns (use the formula of the population covariance and correlation)?

B) Let X_i be any i.i.d distributions with a mean $\mathbb{E}(X_i) = 10$ and variance $V(X_i) = (6)^2$. Let $S_n = X_1 + \ldots + X_n$

1) Which distribution follows S_n for a large n and what is their corresponding parameters (expectation and variance)?

2) Compute the probability: $\mathbb{P}(S_n \geq 15)$ for $n = 6$

3) Compute the probability: $\mathbb{P}(S_n \leq 246)$ for $n = 20$