Example Proof

Prove that the language $B = \{0^n1^n | n \geq 0\}$ is not a regular language.

Proof. Assume that B is regular and let p be the pumping length for the language. Choose the string $s = 0^p1^p$ so that $|s| > p$. By the pumping lemma (PL) for regular languages, we can partition $s = xyz$ so that for any $i \geq 0$, $s' = xy^iz \ \in B$. Let's consider three cases for the partition y:

1. The string (partition) y consists only of 0's. Then, $s' = xyyz = xy^2z$ has more 0's than 1's. Clearly $s' \notin B$ and so condition number 1 of the PL is violated. This is a contradiction for our assumption that B is regular.

2. The string (partition) y consists only of 1's. Using the same argument from the previous case, we obtain another contradiction.

3. The string (partition) y consists of 0's and 1's. Then, $s' = xyyz = xy^2z$ may have the same number of 0's and 1's, but some of the 1's will come before some of the 0's and violate membership in the language B. Hence, we have another contradiction with our assumption of regularity.

Therefore, we cannot avoid a contradiction with any possible y partition and conclude that the language B cannot be a regular language. □

Instructions

Use the Pumping Lemma for CFLs to prove that each of the following languages is not context-free.

1

$A = \{0^n\#0^{2n}\#0^{3n} \mid n \geq 0\}$. Hint: consider $s = 0^p\#0^{2p}\#0^{3p}$.

2

$C = \{w \mid$ the number of 1s equals the number of 2s and the number of 3s equals the number of 4s and $\Sigma = \{1, 2, 3, 4\}$. Hint: consider $s = 1^p3^p2^p4^p$.

3

$B = \{w\#t \mid w \text{ is a substring of } t, \text{where } w, t \in \{a, b\}*\}$. Hint: consider $s = a^p b^p \# a^p b^p$.