**COSC 312**

**Homework #5 Answer Key**

# 1

Produce a context-free grammar (CFG) for each of the following languages, assuming $\Sigma = \{0, 1\}$: **(showing just the rules suffices)**

## 1.a

$\{w \mid w$ starts and ends with different symbols$\}$
Example Answer: $(S \to 0A1 \mid 1A0; A \to 0A \mid 1A \mid \epsilon)$

## 1.b

$\{w \mid$ the length of w is an integer multiplier of 3$\}$
Example Answer: $(S \to 0T \mid 1T \mid \epsilon; T \to 0U \mid 1U; U \to 0S \mid 1S)$

## 1.c

$\{ww^R \mid$ i.e., a word followed by that word reversed$\}$
Example Answer: $(S \to 0S0 \mid 1S1 \mid \epsilon)$

# 2

Let $G = (\{S, A, B, C, D, Z\}, (0, 1), R, S)$,
where $R = \{S \to A \mid C \mid Z; A \to 01B \mid 0A \mid \epsilon; B \to 1B \mid 10A; C \to 10D \mid 1C \mid \epsilon; D \to 01C \mid 0D; Z \to 0Z1 \mid \epsilon\}$.

## 2.a

Describe the language L (in English) that is generated by the CFG G.

Answer: L(G) is the language of either an equal number of disjoint 01 and 10 substrings or the language of n 0's followed by n 1's. By disjoint we mean 0110 rather than 010.

## 2.b

Prove that the language L generated by the CFG G is **not** regular. **Hint**: use the fact that regular languages are closed under union and prove that one component of the language is not regular by the P

Answer:

**Prove that the language $L(G)$ is not a regular language.**

*Proof.*

1. Let L(G) be the language generated by the CFG G.
2. Assume L($G_1$) is the language generated by the rule set $\{S \to Z; Z \to 0Z1 \mid \epsilon\}$
3. Assume L($G_2$) is the language generated by the rule set $\{S \to A \mid C; A \to 01B \mid 0A \mid \epsilon; B \to 1B \mid 10A; C \to 10D \mid 1C \mid \epsilon; D \to 01C \mid 0D\}$
4. If we can show that either L($G_1$) or L($G_2$) is not regular, then L(G) would not be regular by the union closure of regular languages.
5. Assume L($G_1$) is regular.
6. Let $p$ be the pumping length of L($G_1$).
7. Suppose we choose $s = 0^p1^p \in L(G_1)$ so that $|0^p1^p| > p$
8. By the pumping lemma, s can be partitioned as $s = xyz$ such that for all $i \geq 0, xy^iz \in L(G_1)$.
9. By condition 3 of the pumping lemma, s must be divided so that $|xy| \leq p$. Therefore y must contain only one or more 0's $\forall s = xyz$.
10. Let $s' = xy^2z \in L(G_1)$ by choosing $i = 2$.
11. $s' = 0^{p+|y|}1^p \notin L(G_1)$

Therefore, by this contradiction $L(G_1)$ is not regular, and subsequently $L(G)$ must not be regular due to the union closure of regular languages. □