COSC 312

Homework Assignment 4

Points: 10 per proof; total = 40

Name: Solutions Netid: ____________________

1. Prove that C = {w | w has an equal number of 0s and 1s} is not regular. Consider the string $s=0^p1^p$.

(Proof) Assume C is regular and let p be the pumping length of C. Choose $s = 0^p1^p \in C$ so that $|0^p1^p| > p$. According to the Pumping Lemma (PL) we know that s can be partitioned into $s = xyz$ such that for all $i \geq 0$, we have $xy^iz \in C$. By condition # 3 of the PL we must also have $|xy| \leq p$. Hence, y must consist only of 0s. It then follows that $xyyz = xy^2z$ has more 0s than 1s so that $xy^2z \notin C$. Hence, s cannot be pumped - a violation of our assumption that C is regular. By this contradiction, we conclude that the language C is not regular.

(next page)

2. Prove that F = {ww | w is a string from {0,1}*} is not regular. Consider the string $s=0^p10^p1$. Note that {0,1}* is any string (including the empty string) containing any number of 0s and 1s (in any order).

(Proof) Assume F is regular and let p be the pumping length of F. Choose $s = 0^p10^p1 \in F$, where $w = 0^p1$. Clearly, $|0^p10^p1| > p$. According to the Pumping Lemma (PL) s can be partitioned into $s = xyz$ such that for all $i \geq 0$, we have $xy^iz \in F$. By condition # 3 of the PL, we must also have $|xy| \leq p$. Therefore, y must consist only of 0s and subsequently $xyyz = xy^2z \notin F$ since the first part of the string s (i.e., substring w) does not equal the second part of the string s. Hence, s cannot be pumped - a violation of our assumption that F is regular. By this contradiction, we conclude that the language F <u>is not regular</u>.

(next page)

3. Prove that $A = \{www | w \text{ is a string from } \{a,b\}^*\}$ is not regular. Consider the string $s=a^pba^pba^pb$.

(Proof) Assume A is regular and let p be the pumping length of A.

Choose $s = a^pb\,a^pb\,a^pb \in A$, where $w = a^pb$. Clearly, $|a^pba^pba^pb| > p$.

According to the Pumping Lemma (PL) s can be partitioned into $s = xyz$ such that for all $i \geq 0$, we have $xy^iz \in A$.

By condition # 3 of the PL, we must also have $|xy| \leq p$.

Therefore, y must contain only a s and subsequently $xyyz = xy^2z \notin A$ since the first part of the string s (i.e, substring w) will have more as than the second and third parts. Hence, s cannot be pumped - a violation of our assumption that A is regular. By this contradiction, we conclude that the language A is not regular.

(next page)

4. Prove that L= $\{0^n1^m0^n \mid m,n \geq 0\}$ is not regular. Consider the string $s=0^p10^p$.

(Proof) Assume L is regular and let p be the pumping length of L. Choose $s = 0^p10^p \in L$ so that $|0^p10^p| > p$. According to the Pumping Lemma (PL) s can be partitioned into $s = xyz$ such that for all $i \geq 0$, we have $xy^iz \in L$. By condition #3 of the PL, we must also have $|xy| \leq p$. Therefore, y must consist only of 0s and subsequently $xyyz = xy^2z \notin L$ since there would be more 0s in the first part of the string (left of the "1") than in the second part of the string (right of the "1"). Hence, s cannot be pumped – a violation of our assumption that L is regular. By this contradiction, we conclude that the language L is not regular.

Alternative argument...

By condition #1 of PL, s can be divided as $s = xyz$, with $x = 0^a, y = 0^b, z = 0^c10^p$, where $b \geq 1$ and $a+b+c = p$. Pumping down (i.e, choosing $i=0$) we have $xy^0z = xz = 0^a0^c10^p$. But $a+c<p$ (because $b \geq 1$) so the number of 0s to the left of the "1" would decrease and hence $xy^0z = xz \notin L$.