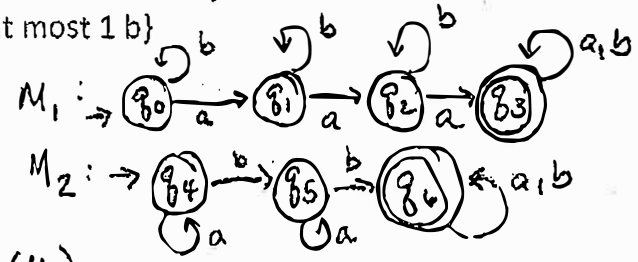
COSC 312
Homework Assignment 2
Points: 1(15), 2(15); total = 30

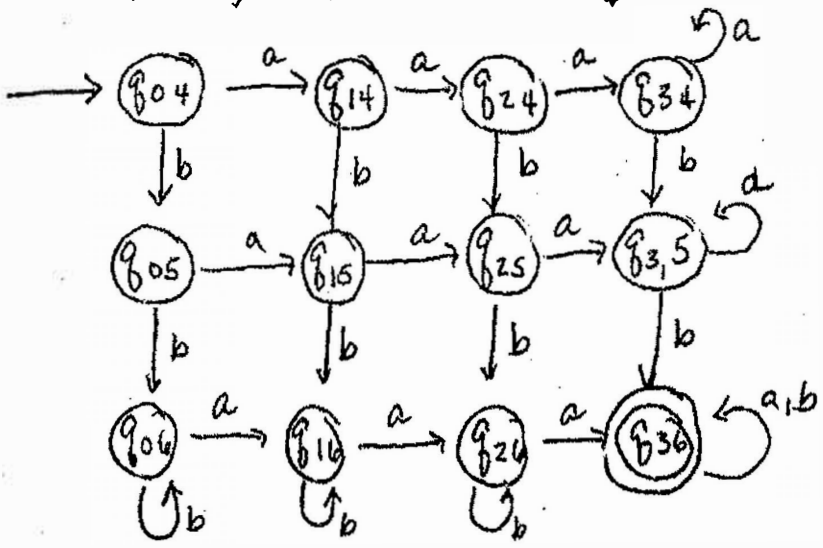
1. Each of the following languages is the intersection of two simpler languages. In each part, construct DFAs for the simpler languages, then combine them using the Cartesian Product of the two DFAs and produce a final state diagram (reducing the number of states where possible) for the complete language. Assume the alphabet $\Sigma = \{a,b\}$.

   a. {w | w has at least 3 a's and at least 2 b's}
   b. {w | w has an even number of a's and 1 or 2 b's}
   c. {w | w starts with an a and has at most 1 b}

a) $L(M_1) = \{w \mid w \text{ has at least 3 a's}\}$
$L(M_2) = \{w \mid w \text{ has at least 2 b's}\}$




$M = M_1 \cap M_2$; $L(M) = L(M_1) \times L(M_2)$

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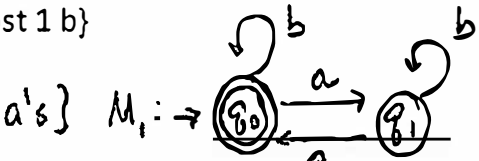
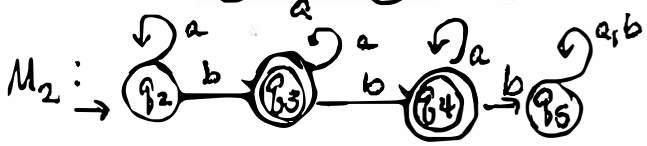
   a. {w | w has at least 3 a's and at least 2 b's}
   b. {w | w has an even number of a's and 1 or 2 b's}
   c. {w | w starts with an a and has at most 1 b}

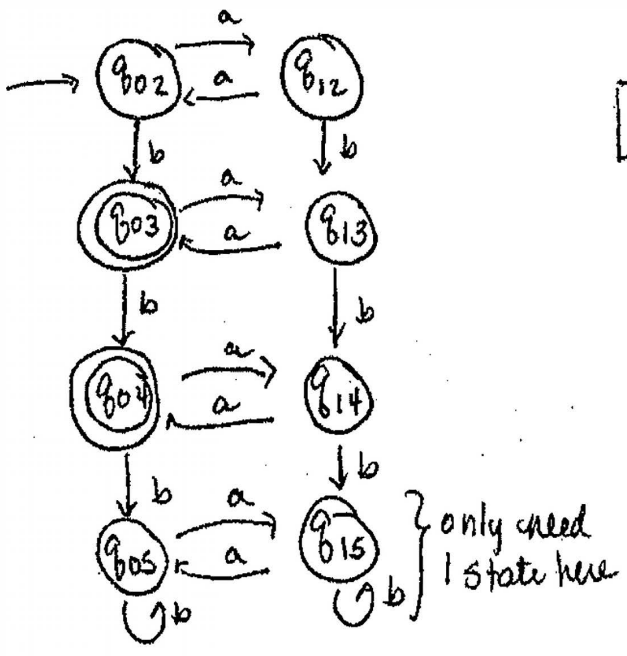
b) $L(M_1) = \{w \mid w \text{ has an even no. of a's}\}$

$L(M_2) = \{w \mid w \text{ has 1 or 2 b's}\}$

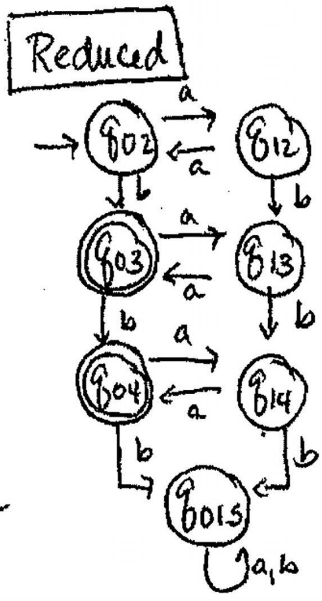



$M = M_1 \cap M_2$; $L(M) = L(M_1) \times L(M_2)$




Reduced




"7 states"

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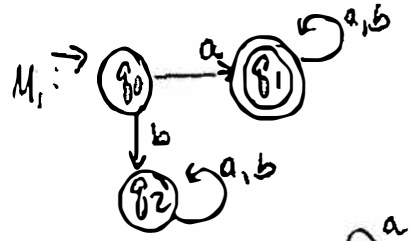
1. Each of the following languages is the intersection of two simpler languages. In each part, construct DFAs for the simpler languages, then combine them using the Cartesian Product of the two DFAs and produce a final state diagram (reducing the number of states where possible) for the complete language. Assume the alphabet $\Sigma = \{a,b\}$.

   a. {w | w has at least 3 a's and at least 2 b's}
   b. {w | w has an even number of a's and 1 or 2 b's}
   c. {w | w starts with an a and has at most 1 b}

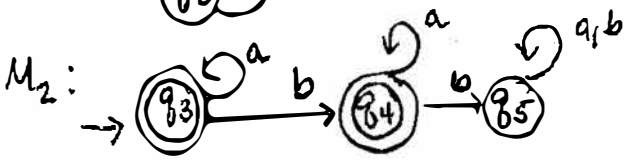
c) $L(M_1) = \{w \mid w \text{ starts with an a}\}$

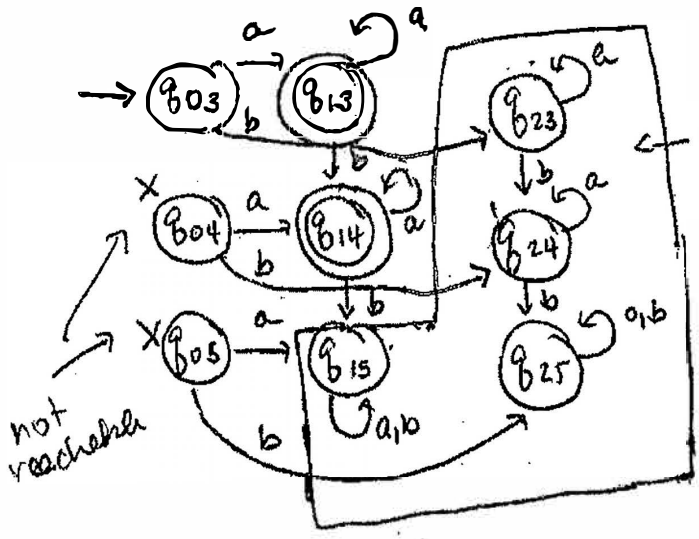
$L(M_2) = \{w \mid w \text{ has at most 1 b}\}$




$M = M_1 \cap M_2$

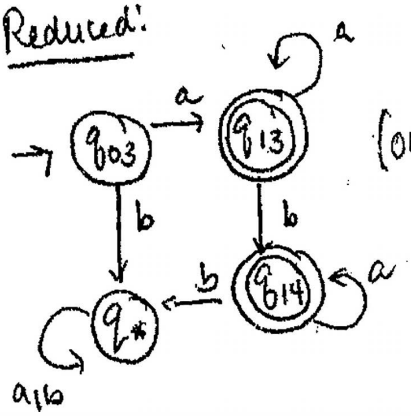
$L(M) = L(M_1) \times L(M_2)$




collapse into a single state, say $q_*$

Reduced:




(only 4 states needed)

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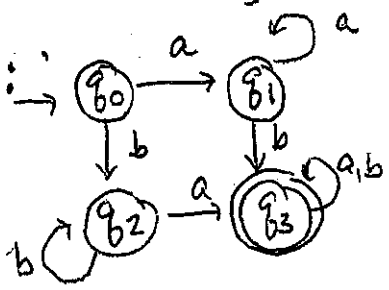
2. Each of the following languages is the complement of a simpler language. In each part, construct a DFA for the simpler language, then use it to produce a final state diagram for the original language. Assume the alphabet $\Sigma = \{a,b\}$.

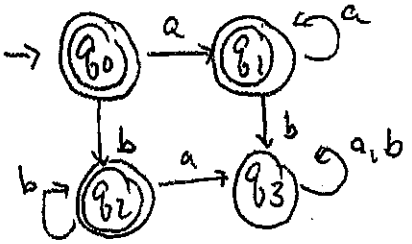
   a. {w | w contains neither the substrings ab nor ba}
   b. {w | w is any string not in a*b*}
   c. {w | w is any string not in $(ab^+)^*$}

a) $\overline{L} = \{w \mid w$ contains the substring ab or substring ba$\}$

$L(\overline{M}) = \overline{L}$; $\overline{M}$:

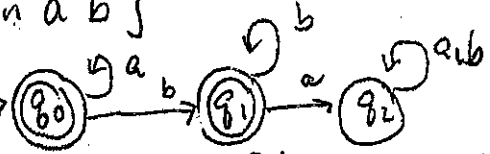



$L(M) = \overline{\overline{L}}$ ⇒ $M$:

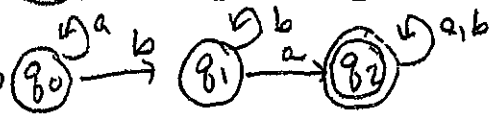



b) $\overline{L} = \{w \mid w$ is a string in $a^*b^*\}$

$L(\overline{M}) = \overline{L}$; $\overline{M}$:



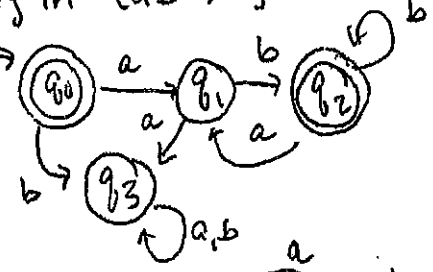

$L(M) = \overline{\overline{L}}$; $M$:




c) $\overline{L} = \{w \mid w$ is a string in $(ab^+)^*\}$

Note: $(ab^+)^* \equiv ab^+ab^+ab^+, \ldots$ $n \in$.

$L(\overline{M}) = \overline{L}$; $\overline{M}$:




$L(M) = \overline{\overline{L}}$;

$M$: