

**Instructions:** Provide your answer to each question (circle only one entry per question) using the scantron sheet that has your name on it. No phones/calculators/laptops are permitted during the exam. You have 75 minutes to complete the exam.

1. Which of the following is the number of (staircase) paths in the  $xy$ -plane from  $(1,2)$  to  $(5,4)$  such that each path is made up of individual steps going one unit to the right (R) or one unit upward (U)?

A) 12    B) 13    C) 14    D) 15

2. Which of the following sets is equivalent to the set

$$(A - B) \cup (A \cap B)?$$

A)  $A^c$     B)  $B$     C)  $B - A$     D)  $A$

3. Suppose  $A=\{1,2,3,4\}$  and  $B=\{2,3,4\}$ . Which of the following sets is  $A \Delta B$ ?

A)  $\{1,3,5\}$     B)  $\{3,4,5\}$     C)  $\{1\}$     D)  $\{1,4\}$

4. Which of the following Diophantine equations does not have a solution over the integers ( $\mathbb{Z}$ )?

- A)  $2x + 4y + 6z = 32$   
 B)  $2x + 6y + 8z = 14$   
 C)  $3x + 6y + 9z = 14$   
 D)  $3x + 6y + 9z = 12$

5. Which of the following is the multiplicative inverse of 5 in  $\mathbb{Z}_7$ ?

A) 2    B) 3    C) 4    D) 5

6. Which of the following propositions is logically equivalent to  $p \rightarrow q$ ?

- A)  $p \wedge \neg q$   
 B)  $\neg p \vee q$   
 C)  $p \vee \neg q$   
 D)  $\neg p \wedge q$

7. Which of the following represents the number of executions of the `cout` statement in the C code fragment below?

```
for (i=0; i < 10; i++) {
    for (j=0; j <= i; j++) {
        for (k=0; k <= j; k++) {
            for (m=0; m <= k; m++) {
                cout << (i*j)+(k*m); }}}}

```

A)  $C(13,4)$     B)  $C(12,4)$     C)  $C(10,4)$     D)  $C(11,4)$

8. Which rule of inference is represented by the logical implication  $q \rightarrow (u \wedge s) \wedge q \implies (u \wedge s)$ ?

- A) Modus Tollens  
 B) Modus Ponens  
 C) Conjunctive Simplification  
 D) Disjunctive Amplification

9. Consider the quantified open proposition  $\forall x [p(x) \rightarrow q(x)]$ . What would the proposition  $\forall x [q(x) \rightarrow p(x)]$  be called in relation to the first proposition?

- A) converse  
 B) contrapositive  
 C) inverse  
 D) complement

10. Which of the following sets is **not** well-ordered?

A)  $\mathbb{Z}^+$     B)  $\{0,1\}$     C)  $\mathbb{Q}$     D)  $\mathbb{N}$

11. Which of the following is equivalent to  $\sum_{k=0}^n \binom{n}{k}$ ?

A)  $n$     B)  $n^2$     C)  $\log_2(n)$     D)  $2^n$

12. How many **unique** permutations of the letters in the word **TWEET** can be created?

A) 20    B) 30    C) 60    D) 120

13. Which of the following is equivalent to the number of ways  $r$  identical objects can be distributed among  $n$  distinct containers?

A)  $\binom{n+r}{r}$     B)  $\binom{n+r-1}{r}$     C)  $\binom{n}{r}$     D)  $\binom{n-1}{r}$

14. Which of the following propositions, that can be shown to be logically equivalent to  $p \rightarrow q$ , is used in the method of proof by contradiction?

- A)  $(p \wedge \neg q) \rightarrow T_0$
- B)  $(p \wedge q) \rightarrow F_0$
- C)  $(p \vee \neg q) \rightarrow T_0$
- D)  $(p \wedge \neg q) \rightarrow F_0$

15. Which of the following open propositions is the correct negation of the proposition  $\exists x \neg p(x)$ ?

- A)  $\forall x p(x)$
- B)  $\forall x \neg p(x)$
- C)  $\exists x p(x)$
- D) none of the above

16. The rule of inference that specifies that  $p(c)$  is true for a randomly selected  $c$  in the universe, if  $\forall x p(x)$  is true, is also known as the \_\_\_\_\_.

- A) Rule of Universal Generalization
- B) Rule of Universal Specification
- C) Rule of Existential Definition
- D) Rule of Existential Quantification

17. Which of the following set relations is **not** equivalent to  $A \subseteq B$ ?

- A)  $\bar{B} \subseteq \bar{A}$
- B)  $A \cap B = A$
- C)  $A \cup B = B$
- D)  $\bar{A} \subseteq \bar{B}$

18. The Euclidean Algorithm can be used to compute

- A) the number of positive prime numbers
- B) the greatest common divisor of two integers
- C) the least common divisor of two integers
- D) none of the above

19. The tautology  $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$  is commonly referred to as the

- A) Rule of Conjunction
- B) Biconditional Equivalence
- C) Law of Syllogism
- D) Conditional Law

20. Which of the following is Euler's Totient for a prime integer  $p$ ?

- A)  $p + 1$
- B)  $p$
- C)  $p - 1$
- D)  $p^2$

21. In RSA encryption, the private key  $d$  is chosen as the multiplicative inverse of the public key  $e$  from which of the following sets? Assume  $n$  is the modulus.

- A)  $\mathbb{Z}_{\phi(n)}$
- B)  $\mathbb{Z}_n$
- C)  $\mathbb{Z}_{n-1}$
- D)  $\mathbb{Z}_{ed}$

22. Which of the following is the dual  $S^d$  for the proposition  $S = (p \wedge \neg q) \vee (r \wedge T_0)$ ?

- A)  $(p \wedge \neg q) \wedge (r \wedge T_0)$
- B)  $(p \wedge \neg q) \vee (r \vee T_0)$
- C)  $(p \vee \neg q) \wedge (r \wedge T_0)$
- D)  $(p \vee \neg q) \wedge (r \vee T_0)$

23. How many integer solutions are there for the parameterized Diophantine equation

$$3(x - 5k) + 5(y + 3k) = 1, \text{ for } k \in \mathbb{Z}?$$

- A) none
- B) one
- C) two
- D) infinite number

24. Suppose you have the following two open statements over the universe of real numbers

$$p(x): x \geq 0$$

$$q(x): x^2 - 3 > 0$$

Which of the following quantified open propositions is **not** true?

- A)  $\forall x [q(x) \rightarrow p(x)]$
- B)  $\exists x [p(x) \wedge q(x)]$
- C)  $\exists x [p(x) \rightarrow q(x)]$
- D)  $\exists x [p(x) \vee q(x)]$

25. Which of the following integers is **not** an element of the recursively defined set A shown below?

$$2 \in A, \forall x \in A, [x + 0.2] \in A$$

- A) 1
- B) 2
- C) 3
- D) 4