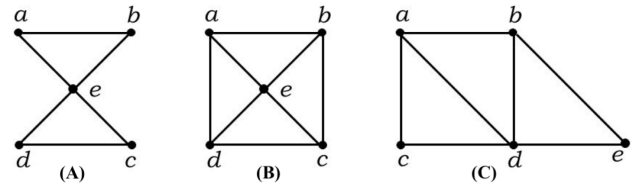


Instructions: This exam is comprised of 25 multiple choice and 2 true/false questions and **1 bonus question**. Provide your answer to each question (circle only one entry per question) using the scantron sheet that has your name on it. No phones or calculators are permitted during the exam.

- Let $G=(V,E)$ be an undirected 5-vertex graph with $V=\{a,b,c,d,e\}$ and $E=\{\{a,b\}, \{b,d\}, \{c,d\}, \{a,c\}, \{a,e\}, \{c,e\}\}$. What is the minimum number of edges for the shortest Hamiltonian cycle in G ?
 A) 3 B) 4 C) 5 D) 6
- Suppose $G=(V,E)$ is a loop-free undirected graph with 8 vertices and each vertex has the same degree. What degree (of each vertex) guarantees that G has an H-cycle?
 A) 1 B) 2 C) 3 D) 4
- Let $G=(V,E)$ be an undirected 5-vertex graph with $V=\{a,b,c,d,e\}$ and $E=\{\{a,b\}, \{b,d\}, \{c,d\}, \{a,c\}, \{c,e\}, \{b,e\}\}$. What is the chromatic number of G ?
 A) 2 B) 3 C) 4 D) 5
- If $G=(V,E)$ is an undirected graph with n vertices and chromatic polynomial given by $P(G, \lambda) = \lambda(\lambda - 1)(\lambda - 2) \cdots (\lambda - n + 1)$. What is the chromatic number of the graph G ?
 A) n^2 B) n C) $n-1$ D) $\log_2 n$
- If $G=(V,E)$ is an undirected 5-vertex graph with $V=\{a,b,c,d,e\}$ and $E=\{\{a,b\}, \{b,c\}, \{c,d\}, \{d,e\}\}$. Which of the following is the chromatic polynomial of G ?
 A) $\lambda(\lambda - 1)^4$ B) $\lambda(\lambda - 1)^3$ C) λ^5 D) $(\lambda - 1)\lambda^4$
- Which of the following is the correct time complexity for computing $h(x) = 1^x + 2^x + 3^x + \cdots + n^x$?
 A) n^x B) $2^{(n-x)}$ C) $n^{(x+1)}$ D) $n + x$
- (True/False) For finite sets A and B , an onto function $f: A \rightarrow B$ requires $|A| < |B|$.
 A) true B) false

- Consider the following 5-vertex connected, undirected, and planar graphs (labeled A through C). Which of the statements below is correct?



- A) Only graph (A) has an Euler circuit
 B) Only graph (B) has an Euler circuit
 C) Graphs (A) and (B) have Euler circuits
 D) Graphs (B) and (C) have Euler circuits
- For n distinct objects, an arrangement (with repetitions allowed) of size 4 can be obtained in how many ways?
 A) n^2 B) n^4 C) 4^n D) $\log_2(4n)$
 - Let $A = \{1,2,3,4,5,6\}$. Which of the following is **not** in the power set of A ?
 A) $\{1,4,5\}$ B) $\{3,5\}$ C) $\{2,6\}$ D) $\{1,2,8\}$
 - Assuming $A \cap B \cap C \neq \emptyset$, which statement below is **false**?
 A) $A \Delta B \supseteq (A - B) - C$
 B) $A \Delta C \supseteq (A - C) - B$
 C) $A \Delta B \subseteq (B - A) - C$
 D) $B \Delta C \supseteq (C - A) - B$
 - Assuming $A = \{1,2,3,7\}$ and $B = \{2,3,7,8\}$, which of the following is $A \Delta B$?
 A) $\{1,8\}$ B) $\{2,3\}$ C) $\{2,3,7\}$ D) $\{1\}$
 - Suppose $a, b \in \mathbb{Z}^+$ and $s, t \in \mathbb{Z}$. If $as + bt = 14$, which of the following integers cannot be the $\gcd(a,b)$?
 A) 1 B) 2 C) 3 D) 7
 - If the Fiedler vector for an 8 by 8 Laplacian matrix associated with a graph $G=(V,E)$ has 4 negative components, how many optimal partitions of the vertices of G are possible?
 A) 16 B) 8 C) 4 D) 2

15. Suppose you have the two sets $A=\{2,3,4,5\}$ and $B=\{w,x,y,z\}$. Which of the following is **NOT** an onto function from A to B?

- A) $\{(3,x), (2,y), (5,w), (4,z)\}$
- B) $\{(2,z), (5,y), (4,w), (3,x)\}$
- C) $\{(4,y), (5,w), (2,x), (3,z)\}$
- D) $\{(5,y), (4,y), (2,x), (3,x)\}$

16. Which of the following time complexities dominates the other three as $n \rightarrow \infty$? Note: *dominates* refers to taking much longer runtime.

- A) $O(n^2)$ B) $O(2^n)$ C) $O(n \log_2 n)$ D) $O(n)$

17. Scheduling the minimum number of taxis for scheduled rides, creating seat assignments given constraints on occupants, and designing timetables for course offerings to minimize conflicts are all applications of _____?

- A) finding Hamiltonian paths
- B) graph partitioning
- C) computing PageRank
- D) graph coloring

18. Which of the following is **not a valid** chromatic polynomial $P(G, \lambda)$ for a graph $G=(V,E)$ with $|V|=3$?

- A) $\lambda^3 - 2\lambda^2 + 1$
- B) $\lambda^3 - 2\lambda^2 + \lambda$
- C) $\lambda^3 - \lambda^2$
- D) $\lambda^3 - 3\lambda^2 + 2\lambda$

19. A graph $G=(V,E)$ is nonplanar if it contains a subgraph that is homeomorphic to _____?

- A) K_3 B) K_3^+ C) $K_{3,3}$ D) $K_{2,3}$

20. Suppose the graph $G=(V,E)$ is connected and planar with $|V|=50$. If the dual graph G^d has 12 vertices, what is $|E|$? That is, how many edges does the graph G have?

- A) 38 B) 48 C) 50 D) 60

21. Which of the following is a directed graph that has a Hamiltonian path?

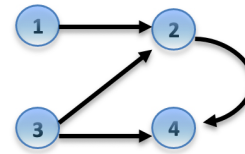
- A) $K_{3,3}$ B) K_5^+ C) Q_3 D) K_4

22. Which of the following is the correct expression for the number of times that `count++` is executed in the code fragment below?

```
count=0;
for (i=0; i<=n; i++) {
    for (j=0; j<=i; j++) {
        count++;
    }
}
```

- A) $\binom{n+1}{2}$ B) $\binom{n+2}{2}$ C) $\binom{n}{2}$ D) $\binom{n-1}{2}$

23. Which of the nodes (pages) of the 4-vertex directed Web graph $G=(V,E)$ below would have the highest PageRank?



- A) 1 B) 2 C) 3 D) 4

24. What value for x is needed to complete the normalized adjacency matrix N for the Web graph in Q22?

$$N = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & x & 0 & x \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

- A) 0 B) 1 C) 0.5 D) 2

25. What is the maximum shortest path (in terms of edges) between any two distinct vertices in a Q_n graph?

- A) $n/2$ B) n C) $n+1$ D) $\log_2 n$

26. Suppose you have the two sets $A=\{2,3,4,5\}$ and $B=\{w,x,y,z\}$. Which of the following is **NOT** a 1-to-1 function from A to B?

- A) $\{(3,x), (2,y), (5,w), (4,z)\}$
- B) $\{(2,z), (5,y), (4,w), (3,z)\}$
- C) $\{(4,y), (5,w), (2,x), (3,z)\}$
- D) $\{(5,y), (4,w), (2,z), (3,x)\}$

27. (True/False) Every cycle in an undirected graph $G=(V,E)$ is also a circuit.

- A) true B) false

28. (Bonus) What is the degree of each vertex in a graph $G=(V,E)$ that is 6-regular?

- A) 6 B) 12 C) 18 D) 36