1. A drama club is holding tryouts for a spring play. With six men and eight women auditioning for the leading male and female roles, how many ways can the director cast his leading couple?
(3 points)
   A. 12
   B. 24
   C. 36
   ✓ D. 48

2. During a local campaign, eight Republican and five Democratic candidates are nominated for president of the school board. If the president is to be one of the candidates, how many possibilites are there for the eventual winner? *(3 points)*

A. 1

B. 8

✓ C. 13

D. 40

3. A local business has a snack shop with a menu consisting of six kinds of muffins, eight kinds of sandwiches, and five different beverages (hot coffee, hot tea, iced tea, cola, and orange juice). Suppose you are asked to get a friend a lunch comprised of a muffin and a hot beverage or a sandwich and a cold beverage. How many lunches could you choose from?

(3 points)

A. 12

B. 24

✓ C. 36

D. 48

1. Which of the following is equivalent to the sum $\binom{3}{0}+\binom{3}{1}+\binom{3}{2}+\binom{3}{3}$?
(3 points)

A. 3^2

✓ B. 2^3

C. 3^4

D. 2^4

2. How many times would the function **`fxn()`** function be called in the code below?

```
for(i=0;i<10;i++){
    for(j=0;j<=i;j++){
        for(k=0;k<=j;k++){
           fxn(i,j,k); }}}
```

(3 points)

A. $\binom{11}{3}$

B. $\binom{9}{3}$

✓ C. $\binom{12}{3}$

D. $\binom{10}{3}$

3. How many times would the function **`fxn()`** function be called in the code below?

```
for(i=0;i<10;i++){
    for(j=0;j<10;j++){
        for(k=0;k<10;k++){
           fxn(i,j,k); }}}
```

(3 points)

A. $\binom{10}{3}$

B. $\binom{12}{3}$

✓ C. 1000

D. 999

1. Which of the following propositions is the converse of $\neg p \rightarrow q$?
(3 points)
   A. $\neg q \rightarrow p$
   B. $\neg q \rightarrow \neg p$
   ✓ C. $q \rightarrow \neg p$
   D. $p \rightarrow \neg q$

2. Which of the following propositions is a **tautology**?
(3 points)
   A. (p OR q) AND (NOT p)
   ✓ B. (p OR q) OR (NOT p)
   C. (p AND q) AND (NOT q)
   D. (p OR q) AND (NOT q)

3. Which of the following is **true** given the following propositions p: t →u and q: ¬t OR u.
(3 points)
   A. p does not logically imply q
   B. q does not logically imply p
   ✓ C. p and q are logically equivalent
   D. none of the above are true

1. Name the rule of inference that validates the following argument:

"If Ron's computer program is correct, then he'll be able to complete his computer science assignment in at most two hours. It takes Ron over two hours to complete his computer science assignment. Therefore Ron's computer program is not correct."

(3 points)

A. Modus Ponens

✓ B. Modus Tollens

C. Disjunctive Syllogism

D. Conjunction

2. What type of argument is reflected in the narrative below?

"A sufficient condition for Sally to win the golf tournament is that her opponent Meg not sink a birdie on the last hole. Sally won the golf tournament. Therefore Meg did not sink a birdie on the last hole."

(3 points)

A. Contradiction

B. Inverse

✓ C. Converse

D. Conjunction

3. Complete the following logic statement that represents the rule of inference known as *Modus Ponens*: $[p \wedge (p \to q)] \to$
(3 points)

A. p

✓ B. q

C. p $\vee$ q ("p or q")

D. $\neg$q ("not q")

1. Which of the following is a tautology according to the Rule of Inference known as Disjunctive Amplification (or Addition)?
   (3 points)
   A. $p \rightarrow p \wedge q$
   ✓ B. $p \rightarrow p \vee q$
   C. $p \rightarrow p \wedge \neg q$
   D. $p \rightarrow p \vee p$

2. Which of the following is equivalent to the tautology p ^ q → r?
(3 points)

A. p ^ q ^ r → F_0

B. p v q v ¬r → F_0

✓ C. p ^ q ^ ¬r → F_0

D. p v q ^ ¬r → F_0

3. What conclusion can be reached from the following argument?

$p \rightarrow q$
$q \rightarrow t$
$\neg t$

(3 points)

✓ A. $\neg p$
B. p
C. $\neg q$
D. $p \wedge \neg t$

1. Which of the following is the negation of the open statement $\forall x\,[p(x) \wedge \neg q(x)]$?

(3 points)

A. $\forall x\,[\neg p(x) \vee q(x)]$

✓ B. $\exists x\,[\neg p(x) \vee q(x)]$

C. $\exists x\,[\neg p(x) \wedge q(x)]$

D. $\exists x\,[p(x) \vee \neg q(x)]$

2. Let p(x,y) denote the open statement "x divides y" where the universe for x and y is all positive integers and "divides" means "divides evenly". Which of the following statements is **false**?

(3 points)

A. $p(3,27)$

B. $\forall x\, p(x,0)$

C. $\forall y\, p(1,y)$

✓ D. $\forall x \forall y\, p(x,y)$

3. Suppose you have the following open statements: p(x): $x^2-8x+15=(x-3)(x-5)=0$ and q(x): x is odd. Which of the following statements is **false**?

(3 points)

A. $\exists x\,[q(x) \rightarrow p(x)]$

B. $\exists x\,[p(x) \rightarrow q(x)]$

✓ C. $\forall x\,[q(x) \rightarrow p(x)]$

D. $\forall x\,[\neg q(x) \rightarrow \neg p(x)]$

1. Suppose you have sets A and B defined by A={1,2,3,4} and B= {0,1,2,3}. Which one of the following set relations is true? *(3 points)*

A. $A \subset B$

B. $B \subset A$

✓ C. $A \triangle B = \{0,4\}$

D. $A \cap B = \{2,3\}$

2. Suppose A={1, 2, 3, 4, 5, 7, 8, 10, 11, 14, 17, 18}. How many subsets of A contain only odd integers? *(3 points)*

   A. 15

   B. 31

   ✓ C. 63

   D. 127

3. Which set below is nonempty? *(3 points)*

   A. All integers x such that 3x+5=9

   B. All real numbers x such that $x^2+6=4$

   ✓ C. All irrational numbers x such that $x^2+4=6$

   D. All natural numbers x such that 2x+7=3

1. The Well-Ordering Principle (WOP) states that every nonempty subset of Z^+contains ____________.
(3 points)
   A. a largest element
   ✓ B. a smallest element
   C. a total order
   D. a dual subset

2. The induction step for the Principle of Mathematical Induction (PMI) insures which of the following logical implications (denoted by →) for an arbitrary integer *k* in Z^+?
(3 points)
   A. p(k+1)→p(k)
   B. p(k)→p(1)
   ✓ C. p(k)→p(k+1)
   D. p(k-1)→p(k)

3. Which one of the following set relations is valid for the recursively defined sets A and B below?

$$\{1 \in A, \forall y \in A, y + 1 \in A\}$$

$$\{2 \in B, \forall z \in B, z \times 2 \in B\}$$

(3 points)

1. A is a proper subset of B.
2. ✓ B is a proper subset of A.
3. The symmetric difference between A and B is the empty set.
4. None of the above.

1. Suppose we want to prove the following open proposition over the natural numbers:

   p(n): $8^n - 3^n$ is a multiple of 5.

   Which of the following would constitute the base case for a proof via PMI?
   (3 points)

   A. p(2)=64-9=55
   B. p(1)=5
   ✓ C. p(0)=0
   D. p(3)=512-27=485

2. For the induction hypothesis of the proof of the open proposition

$p(n)$: $8^n - 3^n$ is a multiple of 5,

we assume $p(k)$ is true for an arbitrary natural number. It then follows that $p(k+1)=8^{k+1}-3^{k+1} = 8\times 8^k - 3\times 3^k$. Which of the following reductions of $p(k+1)$ is needed to complete the proof? *(3 points)*

✓ A. $p(k+1) = 8\times(5j) - 3\times(5m)$, for some integers j and m.

B. $p(k+1) = 8\times(8j) - 3\times(3m)$, for some integers j and m.

C. $p(k+1) = 8^m\times(5j) - 3^j\times(5m)$, for some integers j and m.

D. $p(k+1) = 8 - 3 = 5$, for all $k \geq 0$.

3. Given the proof of the following open proposition over the natural numbers (using PMI)

$p(n)$: $8^n - 3^n$ is a multiple of 5,

which of the following open propositions can also be shown to be a tautology over the natural numbers?

(3 points)

A. $q(n) = 7^n - 3^n$ is a multiple of 5.

B. $r(n) = 6^n - 2^n$ is a multiple of 5.

C. $s(n) = 9^n - 5^n$ is a multiple of 5.

✓ D. $t(n) = 9^n - 4^n$ is a multiple of 5.

1. Which one of the following Diophantine Equations is **not** solvable over the integers?
   (3 points)
   A. $3x + 6y + 9z = 24$
   B. $2x + 8y + 4z = 94$
   C. $4x + 8y + 12z = 68$
   ✓ D. $5x + 10y + 15z = 127$

2. Given that gcd(3,17)=1, which of the following equations is **not** solvable over the integers?
(3 points)

A. $3x + 17y = 1$

B. $6x + 34y = 2$

C. $17x + 3y = 1$

✓ D. $3x + 18y = 17$

3. Suppose you are asked to solve the Diophantine equation $3x + 7y = 41$ over the **natural numbers**. Consider the parameterized equation $41 = 3(-82+7k) + 7(41-3k)$ for all integers k. Which of the following choices for **k** generates an acceptable solution for the Diophantine equation?

(3 points)

A. 11

✓ B. 13

C. 14

D. 15

1. We know that gcd(5,9)=1, i.e., 5 and 9 are "relatively prime". So 5 has a MI in Z_9. Which of the following integers is the MI (Multiplicative Inverse) of 5 in Z_9?
   (3 points)
   - ✓ A. 2
   - B. 3
   - C. 4
   - D. 6

2. For a given prime number p, what is the size of φ(p)? That is, |φ(p)| = ?
(3 points)

   A. p-2
   ✓ B. p-1
   C. p
   D. p+1

3. In RSA encryption, we encrypt the **message** using a public key *e* that is relatively prime with _______ ?
(3 points)

   A. *n*, the modulus (product of two primes)
   B. any integer in Z_n
   ✓ C. φ(n),Euler's totient
   D. None of the above

1. Consider the RSA example below:

Plaintext (P)			Ciphertext (C)		After decryption	
Symbolic	Numeric	P^3	P^3 (mod 33)	C^7	C^7 (mod 33)	Symbolic
S	19	6859	28	13492928512	19	S
U	21	9261	21	1801088541	21	U
Z	26	17576	20	1280000000	26	Z
A	01	1	1	1	01	A
N	14	2744	5	78125	14	N
N	14	2744	5	78125	14	N
E	05	125	26	8031810176	05	E

Sender's computation (Symbolic through P^3 (mod 33)); Receiver's computation (P^3 (mod 33) through Symbolic)

What is the modulus n?
(3 points)

A. 3
B. 7
C. 11
✓ D. 33

2. Consider the RSA example below:

Plaintext (P)			Ciphertext (C)		After decryption	
Symbolic	Numeric	P^3	P^3 (mod 33)	C^7	C^7 (mod 33)	Symbolic
S	19	6859	28	13492928512	19	S
U	21	9261	21	1801088541	21	U
Z	26	17576	20	1280000000	26	Z
A	01	1	1	1	01	A
N	14	2744	5	78125	14	N
N	14	2744	5	78125	14	N
E	05	125	26	8031810176	05	E
Sender's computation				Receiver's computation		

What is the public key e?
(3 points)

✓ A. 3

B. 7

C. 11

D. 33

3. Consider the RSA example below:

Plaintext (P)			Ciphertext (C)		After decryption	
Symbolic	Numeric	P^3	P^3 (mod 33)	C^7	C^7 (mod 33)	Symbolic
S	19	6859	28	13492928512	19	S
U	21	9261	21	1801088541	21	U
Z	26	17576	20	1280000000	26	Z
A	01	1	1	1	01	A
N	14	2744	5	78125	14	N
N	14	2744	5	78125	14	N
E	05	125	26	8031810176	05	E
Sender's computation				Receiver's computation		

What is the private key d?
(3 points)

A. 3
✓ B. 7
C. 11
D. 33

1. Let A={1,2,3} and B={w,x,y,z}. How many onto functions f:A→B are possible?
   (3 points)
   - ✓ A. 0
   - B. 3
   - C. 4
   - D. 12

2. Let A={1,2,3} and B={x,y,z}. Which one of the following functions from A to B are **not** onto?
(3 points)

A. {(1,x),(2,y),(3,z)}

B. {(2,x),(3,y),(1,z)}

✓ C. {(3,y),(2,x),(1,x)}

D. {(3,z),(1,y),(2,x)}

3. Let A={1,2,3} and B={x,y,z}. Which one of the following functions from A to B is not a 1-to-1 correspondence?
(3 points)

A. {(1,x),(2,y),(3,z)}

B. {(2,x),(3,y),(1,z)}

C. {(3,y),(2,z),(1,x)}

✓ D. {(2,z),(1,y),(2,x)}

1. Consider f,g,h : $Z^+ \rightarrow R$ with f(n) = 2n +1, g(n) = n^2-1 and h(n)=1-n for all n in Z^+. Which of the following statements is **false**?
(3 points)

A. h is O(f)

B. f is O(g)

✓ C. g is O(h)

D. h is O(g)

2. What is the runtime complexity of the following C++ code fragment:

```
sum=0; i=n;
while (i > 0) {
   sum++;
   i=i-2; }
```

(3 points)

A. $O(n^2)$

B. $O(\log_2 n)$

C. $O(1)$

✓ D. $O(n)$

3. Which list of runtime complexities is in the correct order from shortest (left) to longest (right)?
(3 points)

✓ A. $(\log_2 n)$, n, $n(\log_2 n)$, n^2, 2^n

B. $(\log_2 n)$, n, n^2, $n(\log_2 n)$, 2^n

C. n, $(\log_2 n)$, $n(\log_2 n)$, n^2, 2^n

D. n, $(\log_2 n)$, n^2, $n(\log_2 n)$, 2^n

1. Which of the following is the appropriate recurrence relation for the time complexity of a recursive function that generates Fibonacci numbers?
(3 points)
   A. $T(n)=T(n-1)+c_1;T(0)=0, T(1)=1$
   B. $T(n)=T(n/2)+c_1; T(0)=0, T(1)=1$
   ✓ C. $T(n)=T(n-1)+T(n-2); T(0)=0, T(1)=1$
   D. $T(n)=T(n/2)+c_1n; T(0)=0, T(1)=1$

2. Which of the following is the appropriate recurrence relation for the time complexity of a recursive function that executes Binary Search? *(3 points)*

   A. $T(n)=T(n-1)+c_1;T(1)=1$

   ✓ B. $T(n)=T(n/2)+c_1; T(1)=1$

   C. $T(n)=T(n-1)+c_1n;T(1)=1$

   D. $T(n)=T(n/2)+c_1n; T(1)=1$

3. Which of the following is the appropriate recurrence relation for the time complexity of a recursive function that executes Merge Sort? *(3 points)*

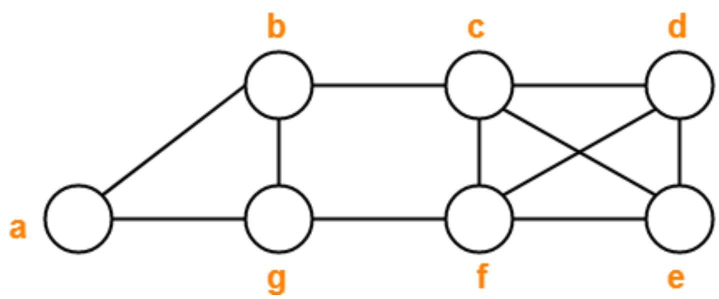
✓ A. $T(n)=2\times T(n/2)+c_1n; T(1)=1$

B. $T(n)=T(n/2)+c_1; T(1)=1$

C. $T(n)=T(n-1)+c_1; T(1)=1$

D. $T(n)=T(n-2)+c_1n; T(1)=1$

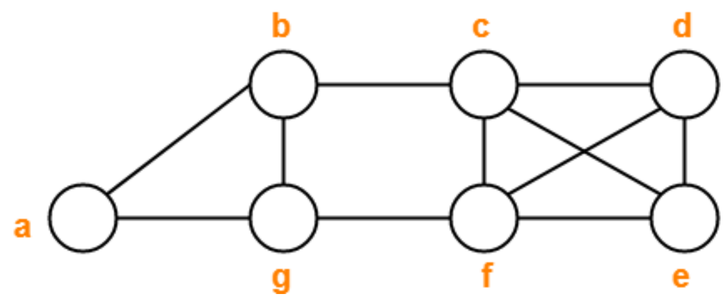
1. Which of the following is **not** a valid open walk in the graph shown below?



(3 points)

1. a→b→c→d→e→f
2. f→d→c→b→g→a
3. e→c→f→d→c→b
4. ✓ g→b→c→f→f→d

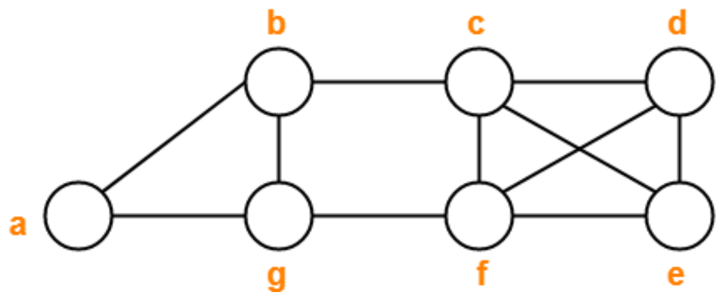
2. Which of the following is **not** a valid trail in the graph shown below?



(3 points)

1. a→b→c→d→e→f
2. ✓ f→d→c→e→d→c
3. e→c→f→d→c→b
4. g→b→c→f→d→e

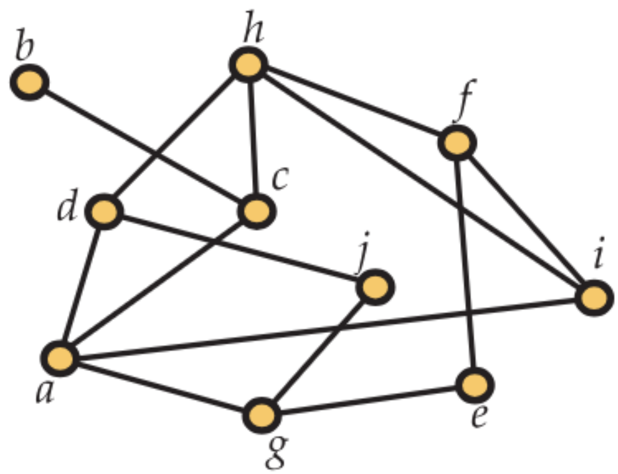
3. Which of the following is **not** a valid path in the graph shown below?



(3 points)

1. a→b→c→d→e→f
2. f→d→c→b→a→g
3. ✓ e→c→f→d→c→b
4. g→b→c→f→d→e

1. Consider the following graph G=(V,E) with |V|=10 and |E|=12.



Which of the following is **not** an **induced** subgraph?

(3 points)

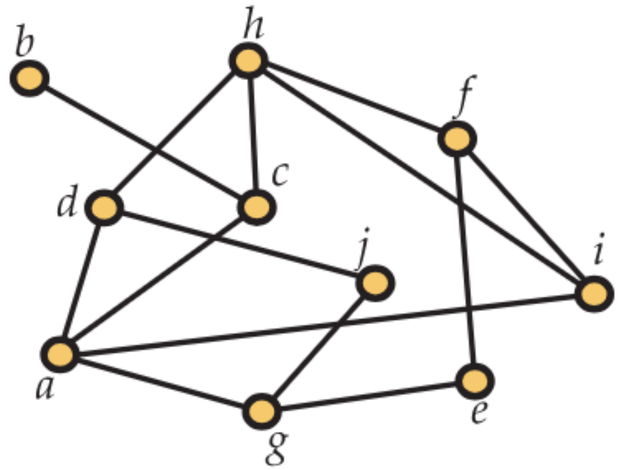
A. {{a,d},{d,j},{g,j},{a,g}}

B. {{a,d},{d,j},{g,j},{a,g},{g,e}}

C. {{a,d},{d,j},{g,j},{a,g},{g,e},{e,f}}

✓ D. None, all are induced subgraphs.

2. Consider the following graph G=(V,E) with |V|=10 and |E|=12.



If you removed vertex ***b*** (and its incident edges) from G, how many **spanning** subgraphs of the original graph G could you produce?
(3 points)

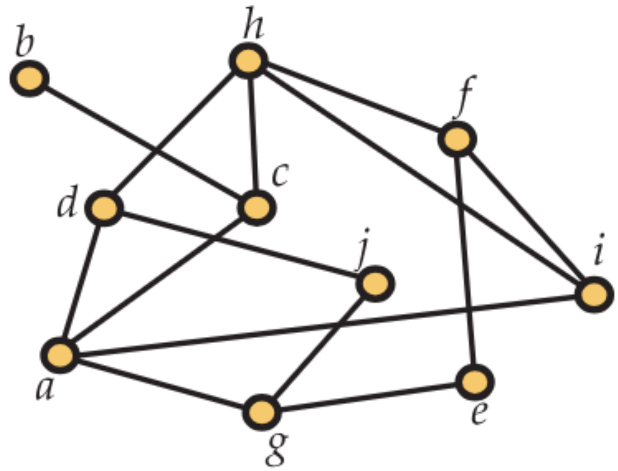
✓ A. 0

B. 1

C. 2

D. 10

3. Consider the following graph G=(V,E) with |V|=10 and |E|=12.



Suppose the subgraph G' is obtained from G by removing vertices ***h*** and ***e*** (and all their incident edges). What is *K*(G')?

(3 points)

✓ A. 1

B. 2

C. 3

D. 8

1. Suppose the graph G=(V,E) is a 4-regular graph and |V|=10. Which of the following is |E|?
(3 points)

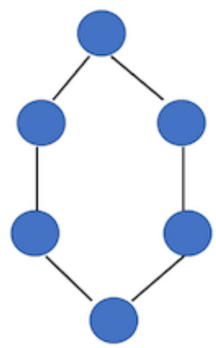
A. 10

✓ B. 20

C. 30

D. 40

2. Suppose three **pendant** vertices are added to the 6-vertex graph below. What is the **maximum number** of vertices of **odd degree** that the resulting graph could have?



(3 points)

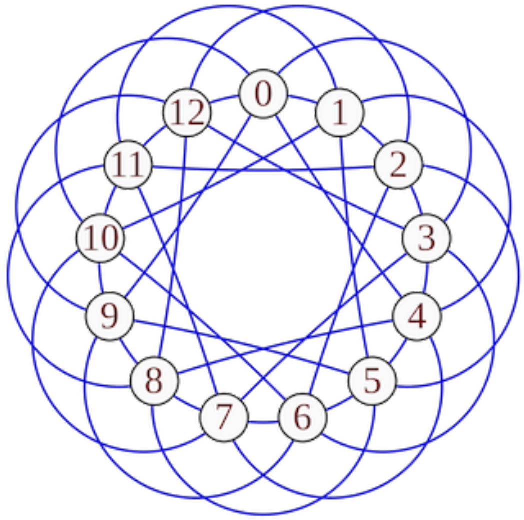
A. 3

B. 4

C. 5

✓ D. 6

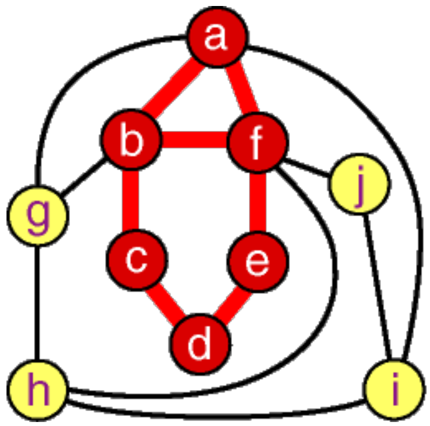
3. Consider the Paley graph G=(V,E) of order 13 below:



Given that |V|=13, which of the following is the correct value for |E|?
(3 points)

A. 26
B. 36
✓ C. 39
D. 48

1. Consider the 10-vertex graph G below. What type of graph property does the 6-vertex subgraph of red vertices and edges have relative to graph G?



(3 points)

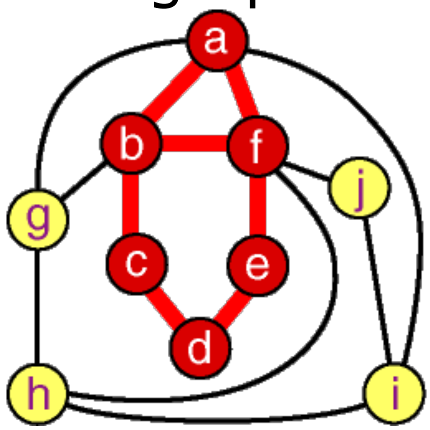
A. spanning

✓ B. induced

C. complete

D. none of the above

2. How many distinct K_3 subgraphs are in the 10-vertex graph G below?



(3 points)

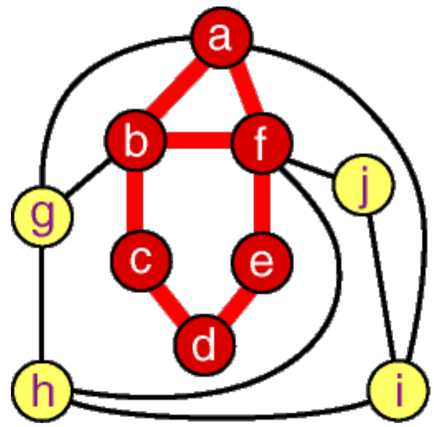
A. 0

B. 1

✓ C. 2

D. 3

3. Within the 10-vertex graph G shown below, consider the 4-vertex subgraph G_1 defined by the vertex set V_1={f,h,i,j}. Which of the following subgraphs (defined by their vertex sets) is <u>isomorphic</u> to G_1?



(3 points)

A. V_2={a,b,e,f}

B. V_3={a,b,g,h}

C. V_4={a,b,f,j}

✓ D. V_5={a,g,h,i}

1. Which of the following graphs is **not** planar?
(3 points)

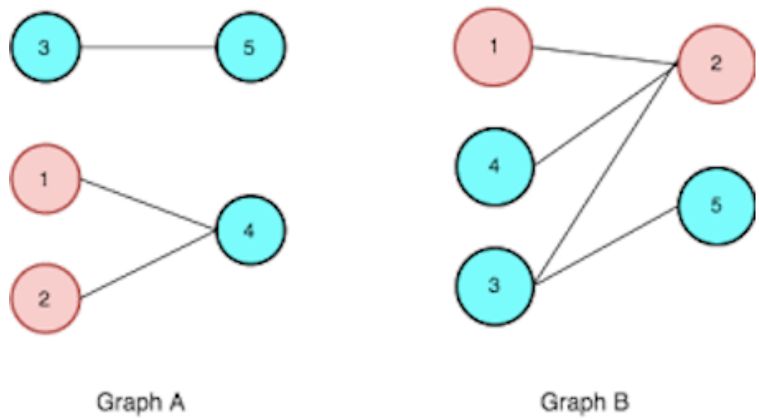
A. K_4

B. $K_{2,3}$

✓ C. $K_{3,3}$

D. Q_3

2. Consider the two graphs A and B below, each having two different vertex subsets of size 2 and 3.



Which of the following statements is true?

(3 points)

A. A and B are both bipartite

B. Only A is bipartite

C. Only B is bipartite

✓ D. Neither A nor B is bipartite

3. Suppose the undirected graph G=(V,E) with |V|=6. If its adjacency matrix A is the following 6×6 matrix

$$\begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \end{pmatrix},$$

which of the following is true?

(3 points)

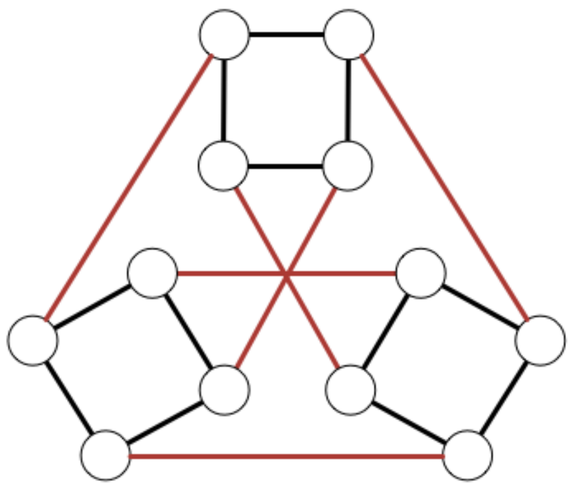
A. G is bipartite and planar

B. G is 6-regular and planar

✓ C. G is bipartite and nonplanar

D. G is not bipartite and planar

1. What is the minimal number of edges composing a cut-set that yields 2 components from the graph below?



(3 points)

A. 2

✓ B. 3

C. 4

D. 5

2. How many regions does a connected planar graph G=(V,E) produce if |V|=50 and |E|=60?

(3 points)

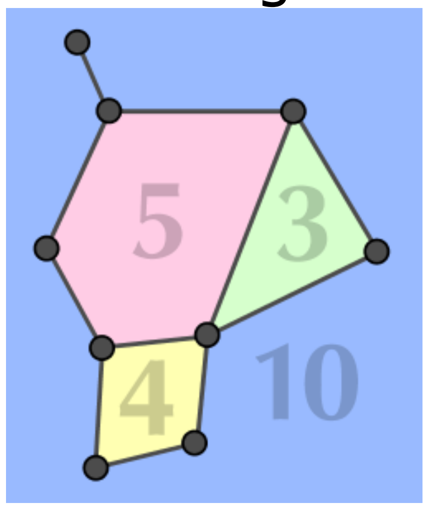
A. 10

B. 11

✓ C. 12

D. 13

3. What is the degree of Region 10 in the planar embedding below?



(3 points)

A. 7

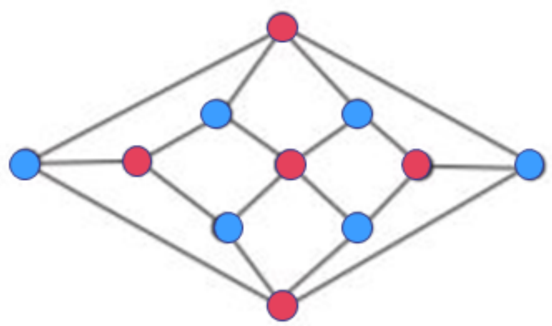
B. 8

C. 9

✓ D. 10

1. Which of the graphs below does **NOT** have a Hamiltonian path?
   (3 points)
   A. K_3
   B. Q_4
   C. K_3^+
   D. K_4^+
   ✓ E. All the above have H-paths

2. Consider the graph G=(V,E) with |V|=11 and |E|=18 below.



Which of the following is **NOT** a property of G?

(3 points)

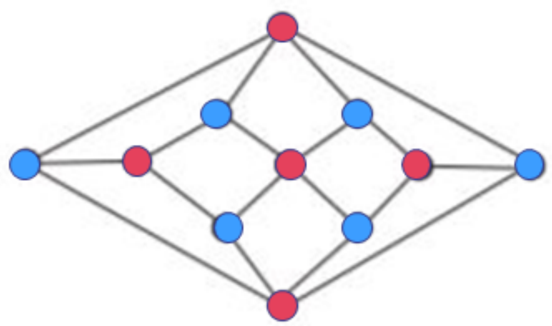
A. It is bi-partite.

B. It is planar with 9 regions.

✓ C. It has a Hamiltonian cycle.

D. It does not have a K_4 subgraph.

3. Consider the graph G=(V,E) with |V|=11 and |E|=18 below.



Which of the following is a property of G?
(3 points)

A. It is 4-regular.

B. It is complete.

✓ C. It has a Hamiltonian path.

D. It has 8 components, i.e, K(G)=8.