

1. Suppose we want to prove the following open proposition over the natural numbers:

$p(n)$: $8^n - 3^n$ is a multiple of 5.

Which of the following would constitute the base case for a proof via PMI?

(3 points)

- A. $p(2) = 64 - 9 = 55$
- B. $p(1) = 5$
- ✓ C. $p(0) = 0$
- D. $p(3) = 512 - 27 = 485$

2. For the induction hypothesis of the proof of the open proposition

$p(n)$: $8^n - 3^n$ is a multiple of 5,
we assume $p(k)$ is true for an arbitrary natural number. It then follows that $p(k+1) = 8^{k+1} - 3^{k+1} = 8 \times 8^k - 3 \times 3^k$. Which of the following reductions of $p(k+1)$ is needed to complete the proof?
(3 points)

- ✓ A. $p(k+1) = 8 \times (5j) - 3 \times (5m)$, for some integers j and m .
- B. $p(k+1) = 8 \times (8j) - 3 \times (3m)$, for some integers j and m .
- C. $p(k+1) = 8^m \times (5j) - 3^j \times (5m)$, for some integers j and m .
- D. $p(k+1) = 8 - 3 = 5$, for all $k \geq 0$.

3. Given the proof of the following open proposition over the natural numbers (using PMI)

$p(n)$: $8^n - 3^n$ is a multiple of 5,

which of the following open propositions can also be shown to be a tautology over the natural numbers?

(3 points)

- A. $q(n) = 7^n - 3^n$ is a multiple of 5.
- B. $r(n) = 6^n - 2^n$ is a multiple of 5.
- C. $s(n) = 9^n - 5^n$ is a multiple of 5.
- ✓ D. $t(n) = 9^n - 4^n$ is a multiple of 5.